Seismic Refraction Method

Koichi Hayashi

Seismic waves

- Body waves

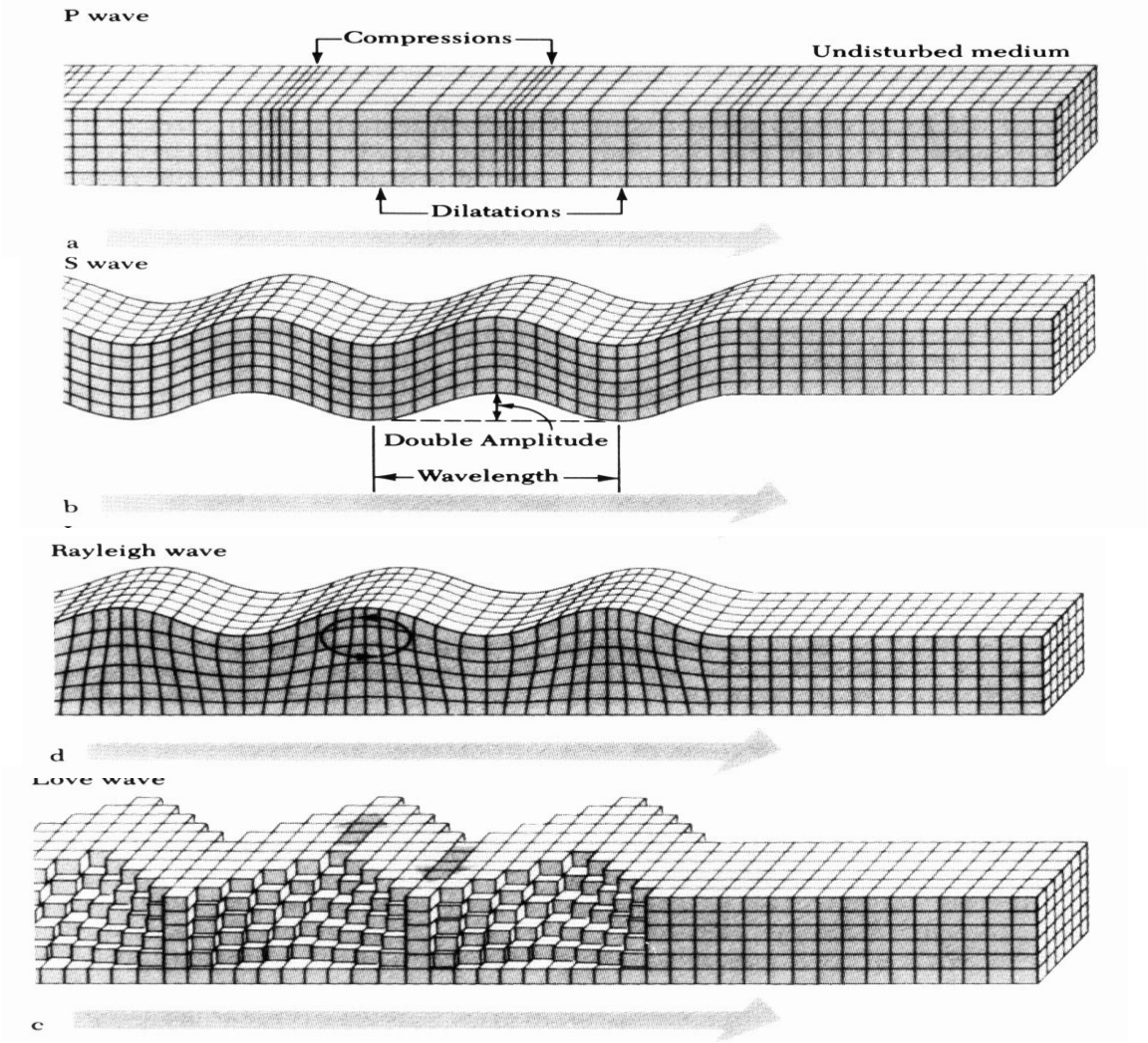
  P-wave

  S-wave

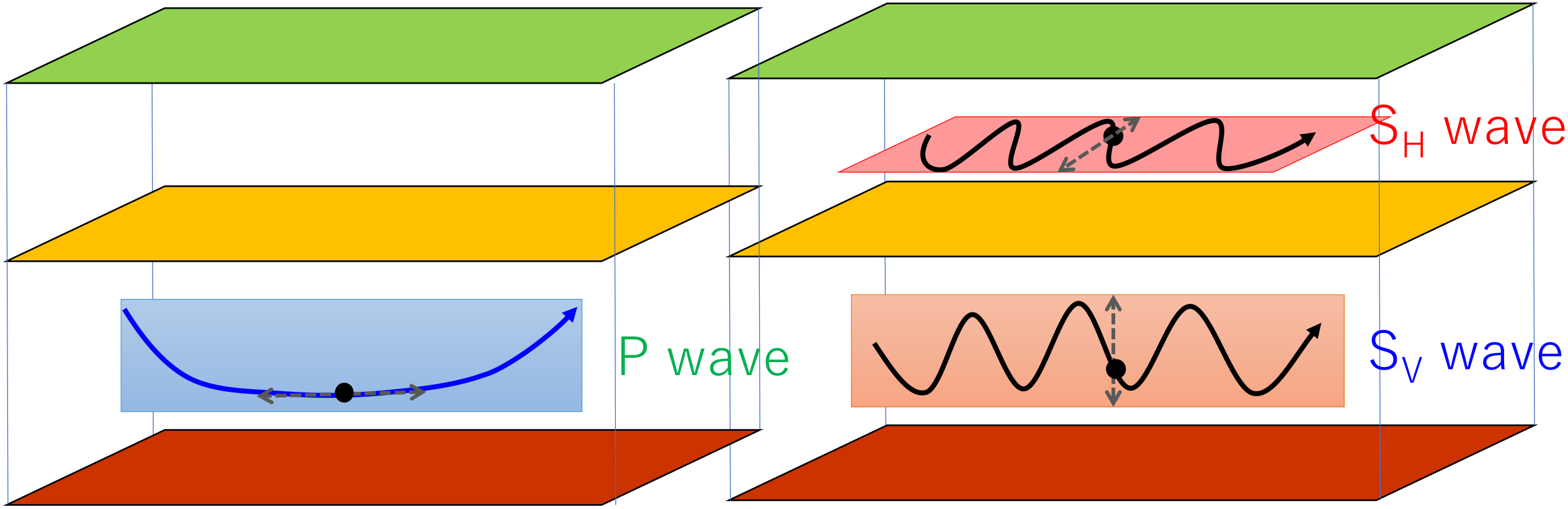
- Surface-waves

  Rayleigh-wave

  Love-wave

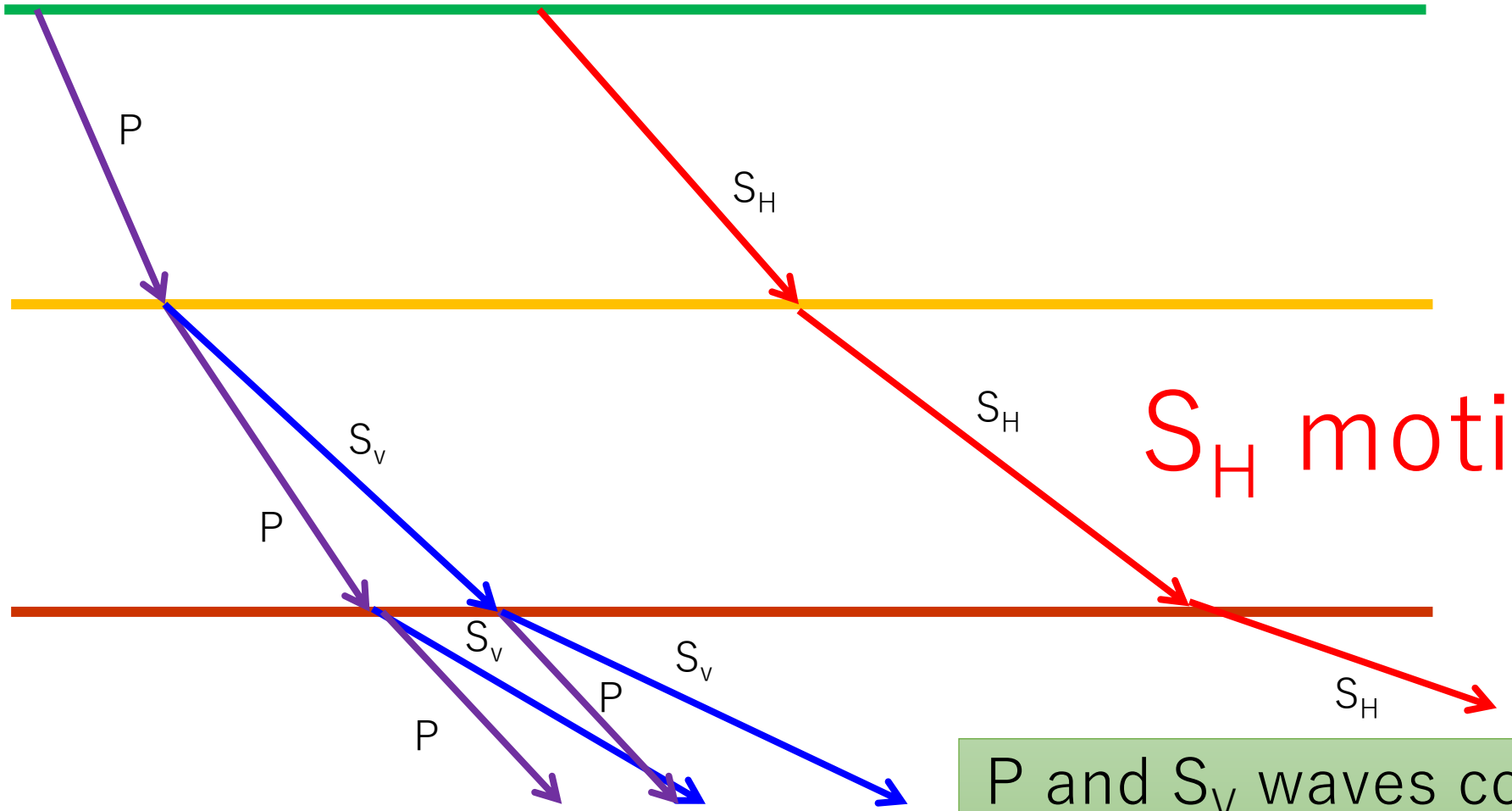


P, S_V and S_H waves in two-dimensional model



P and S_V waves vibrate in the same plane.
S_H waves vibrate perpendicular to P and S_V waves.

P-S_V and S_H waves

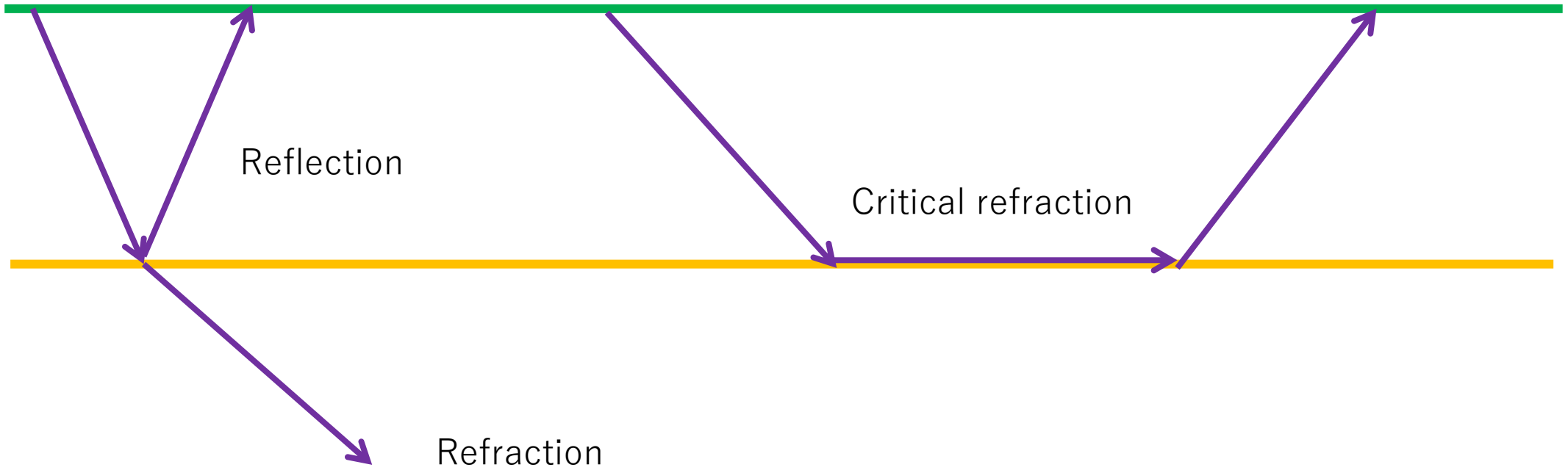


S_H motion

P-S_V motion

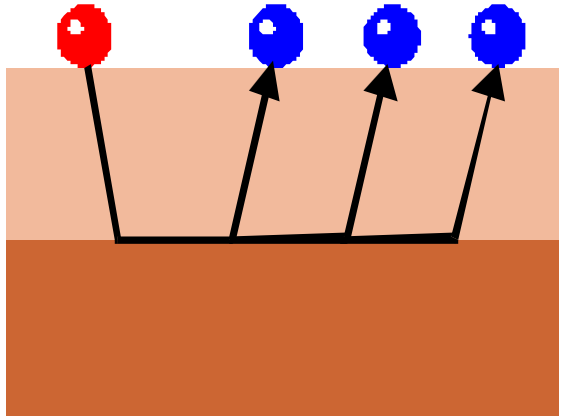
P and S_V waves convert each other.
S_H waves never convert.

Refraction and reflection

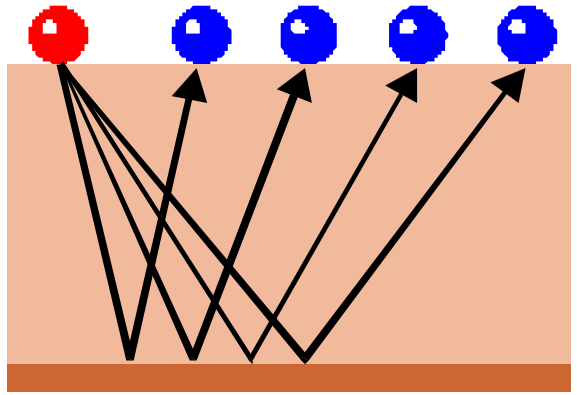


Seismic methods using body waves

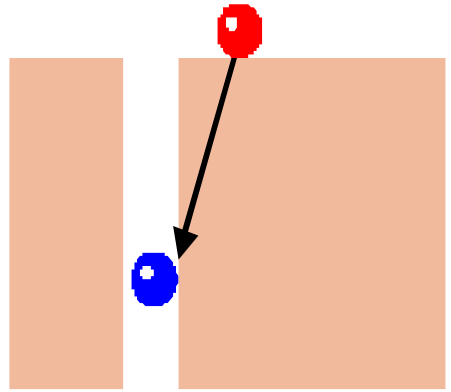
Refraction method



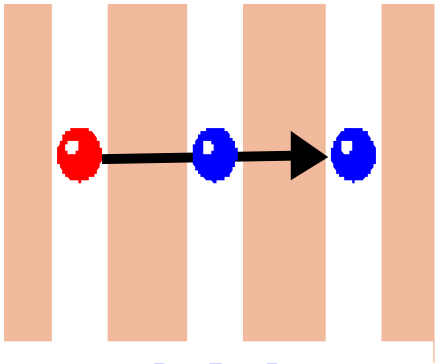
Reflection method



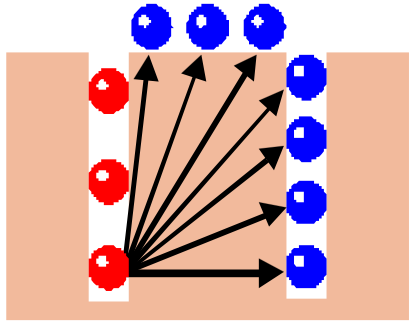
Downhole



Cross-hole



Cross-hole tomography



Sources
Receivers
Ray-path

General geology and applicability of seismic methods

Structure basin

Passive surface wave

Active surface wave

S-wave reflection

Mountainous area

P-wave refraction

Active surface wave

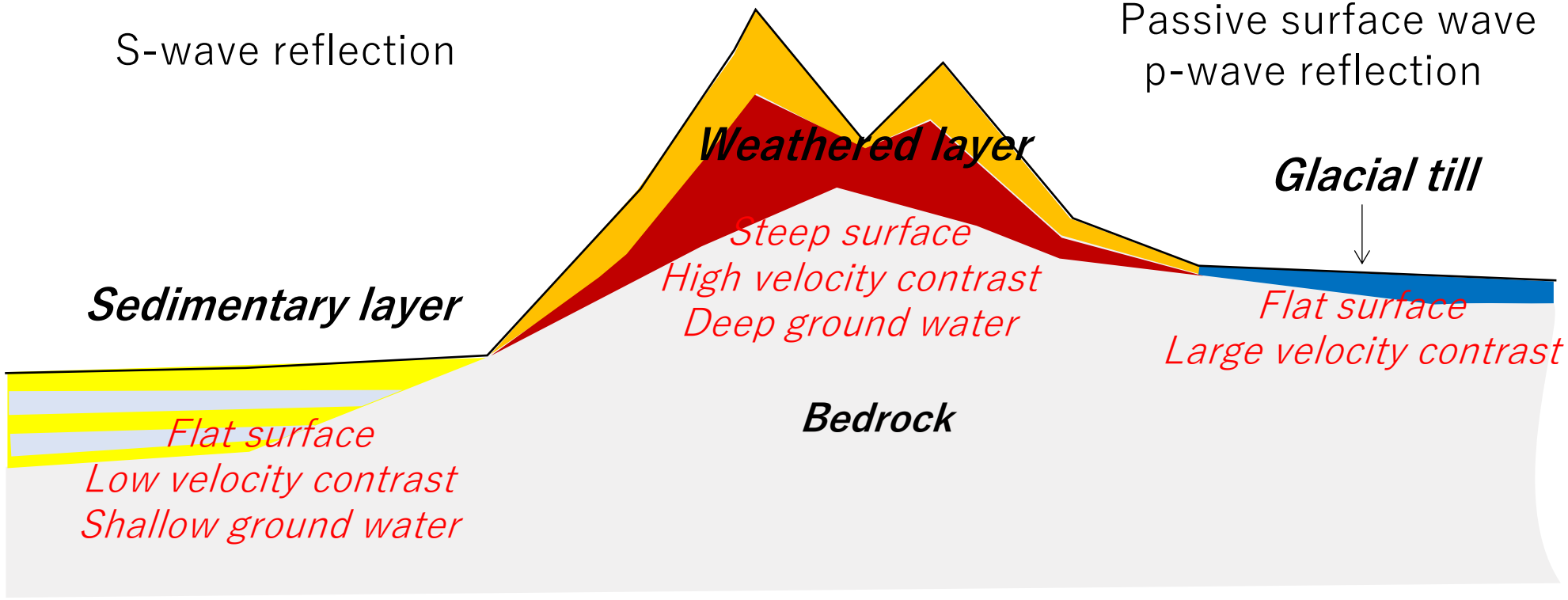
Glacial till

P-wave refraction

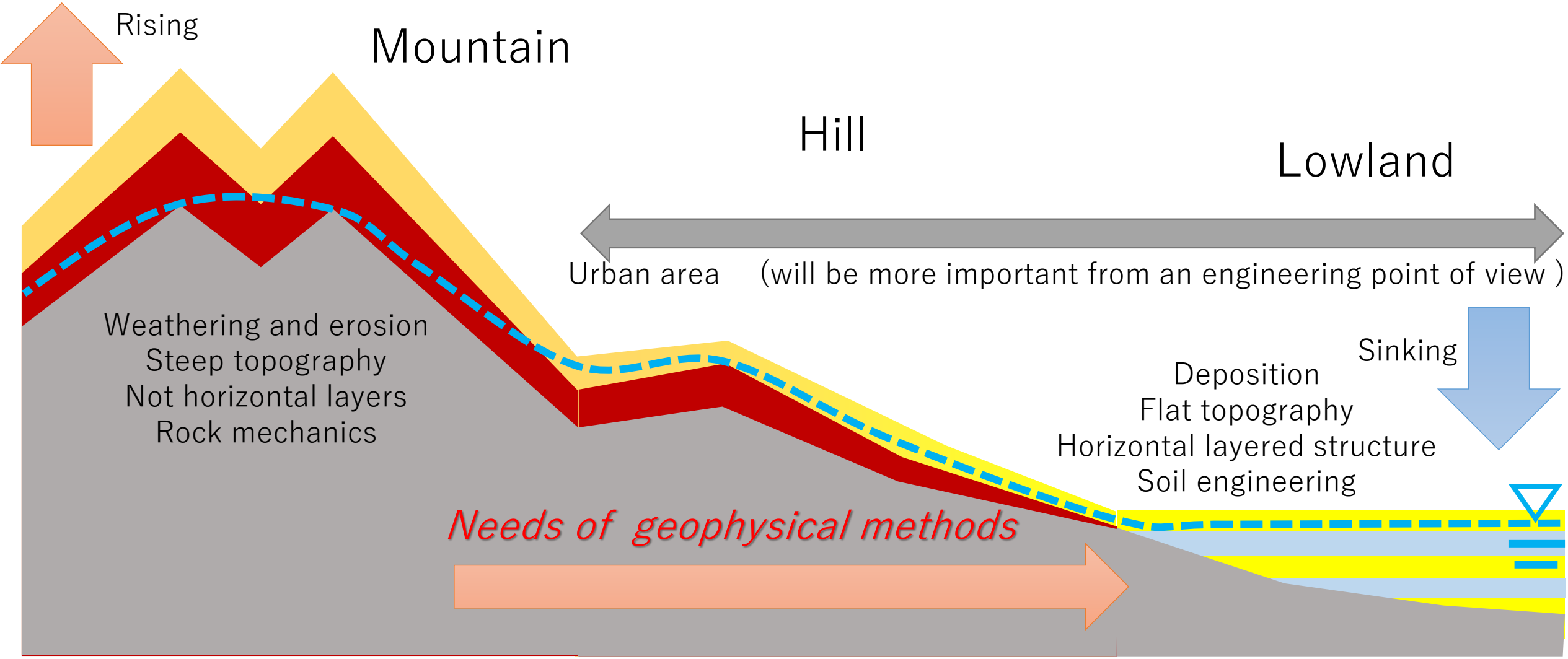
Active surface wave

Passive surface wave

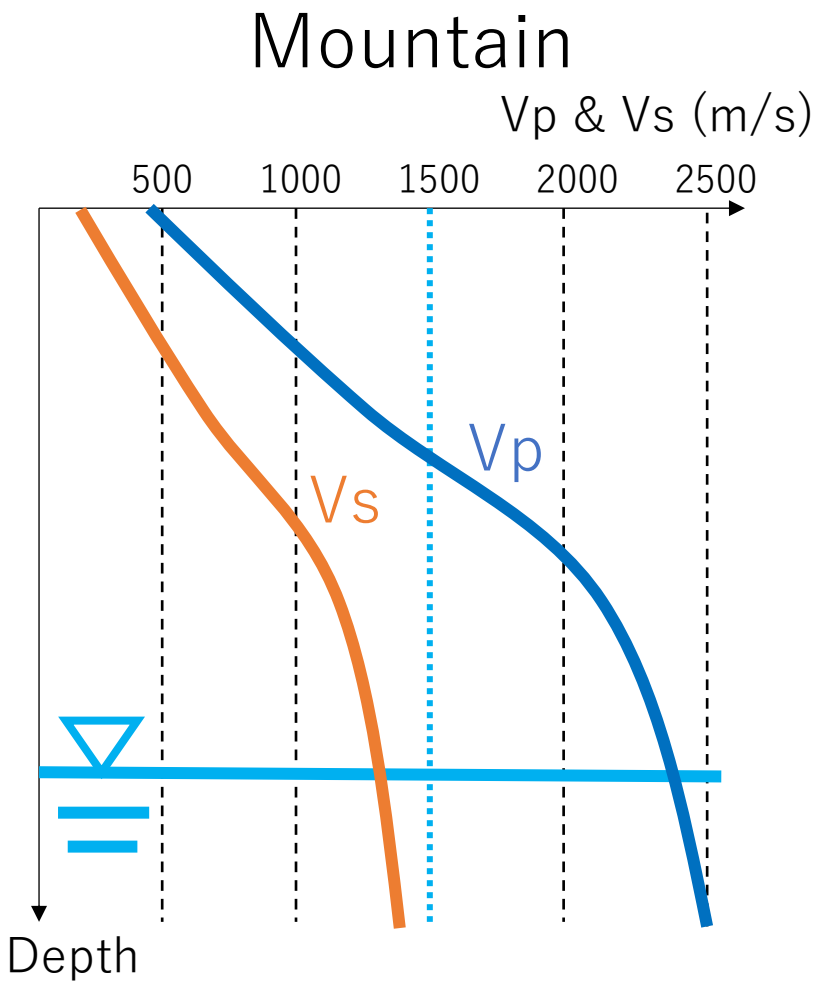
p-wave reflection

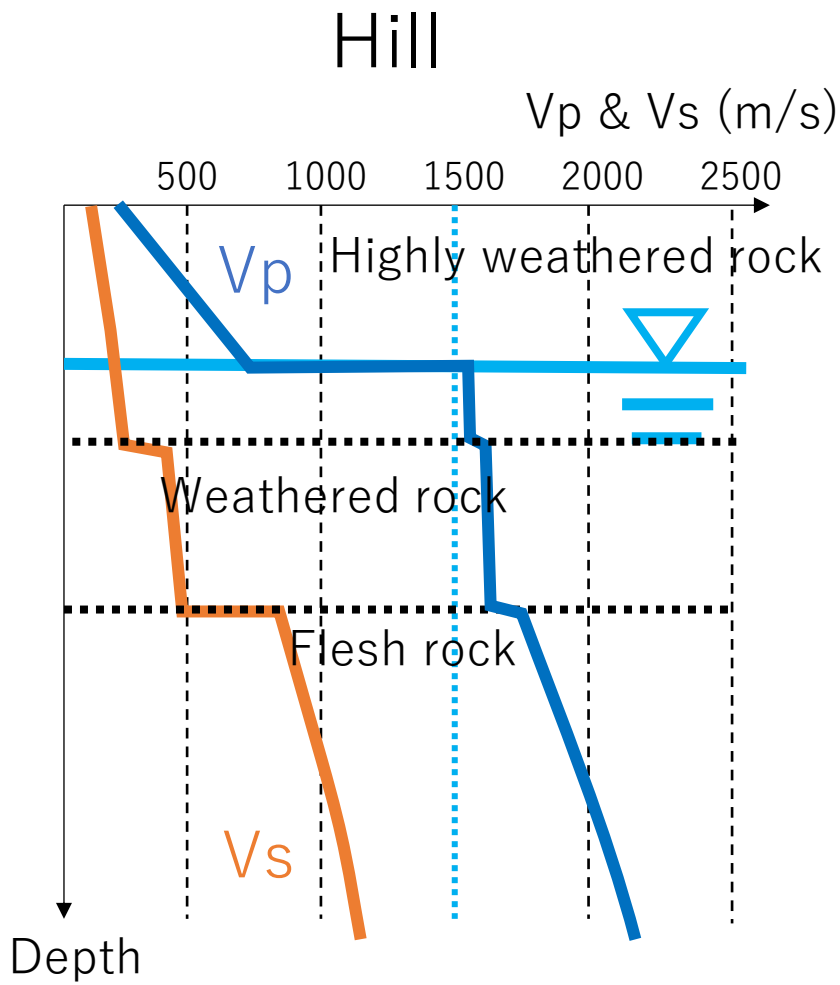


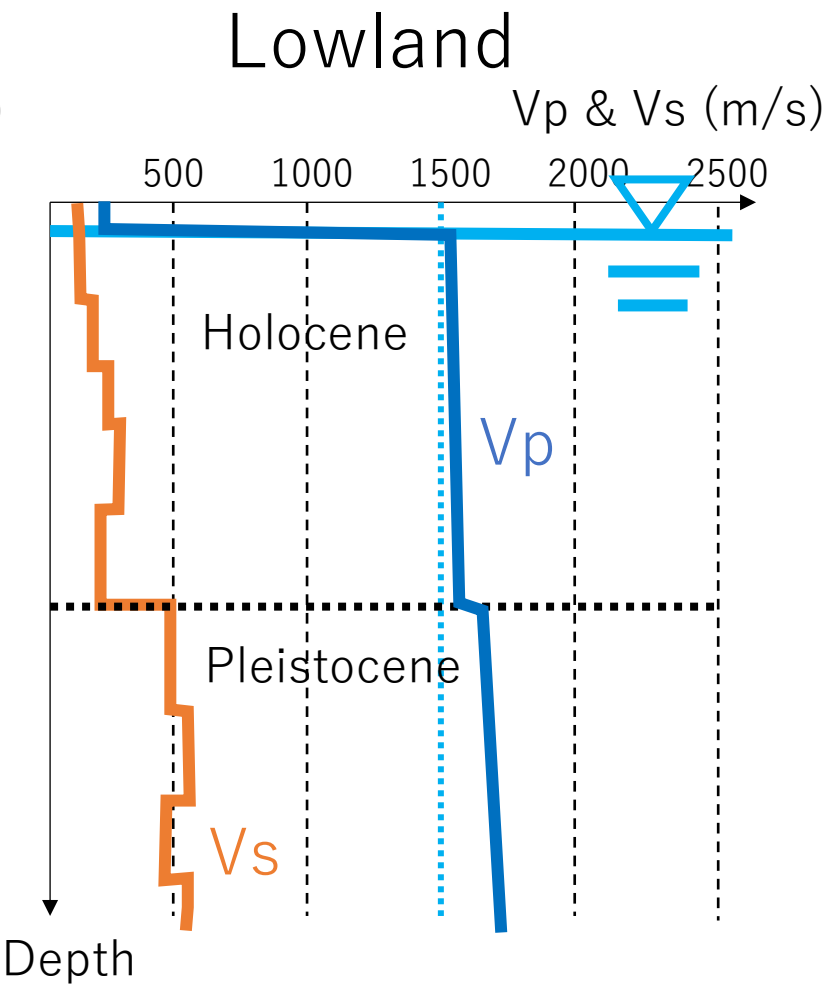
Why S-wave velocity?



Why S-wave velocity?



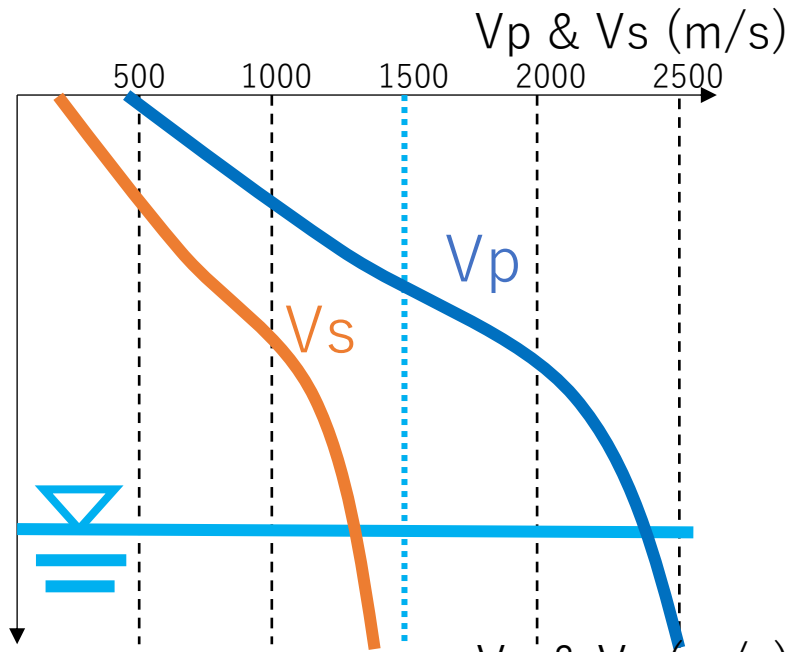




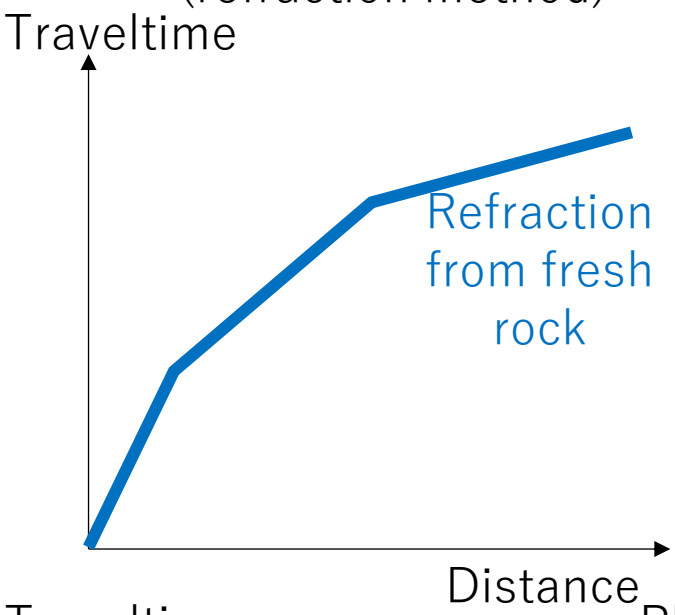
Vp cannot be used as a parameter of stiffness at unconsolidated saturated soil

Why surface wave methods?

Mountain

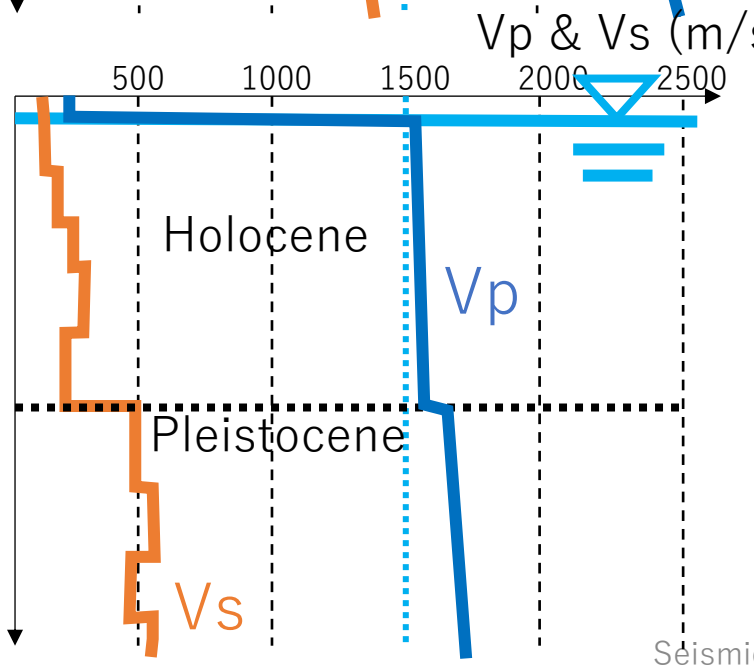


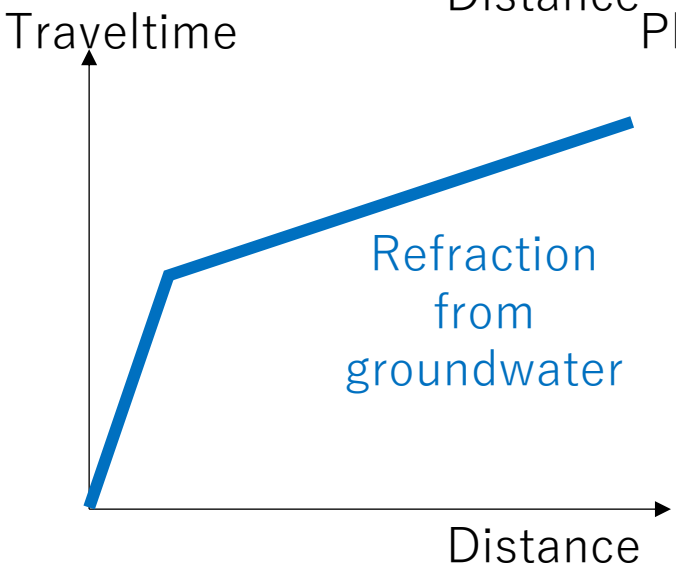
Traveltime curve
(refraction method)

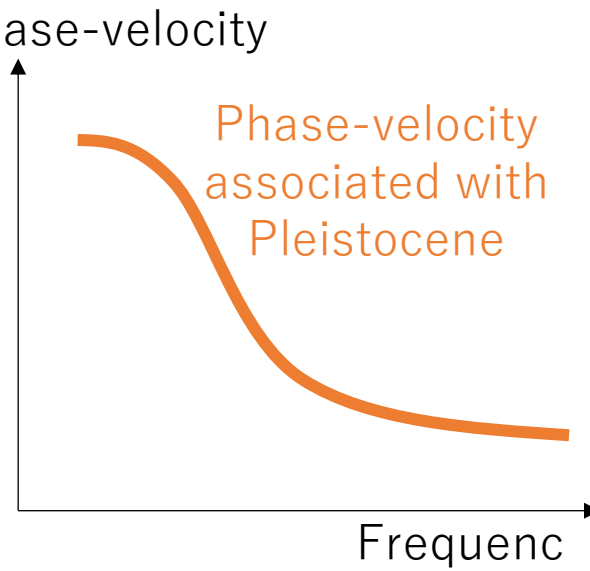


Dispersion curve
(surface wave methods)

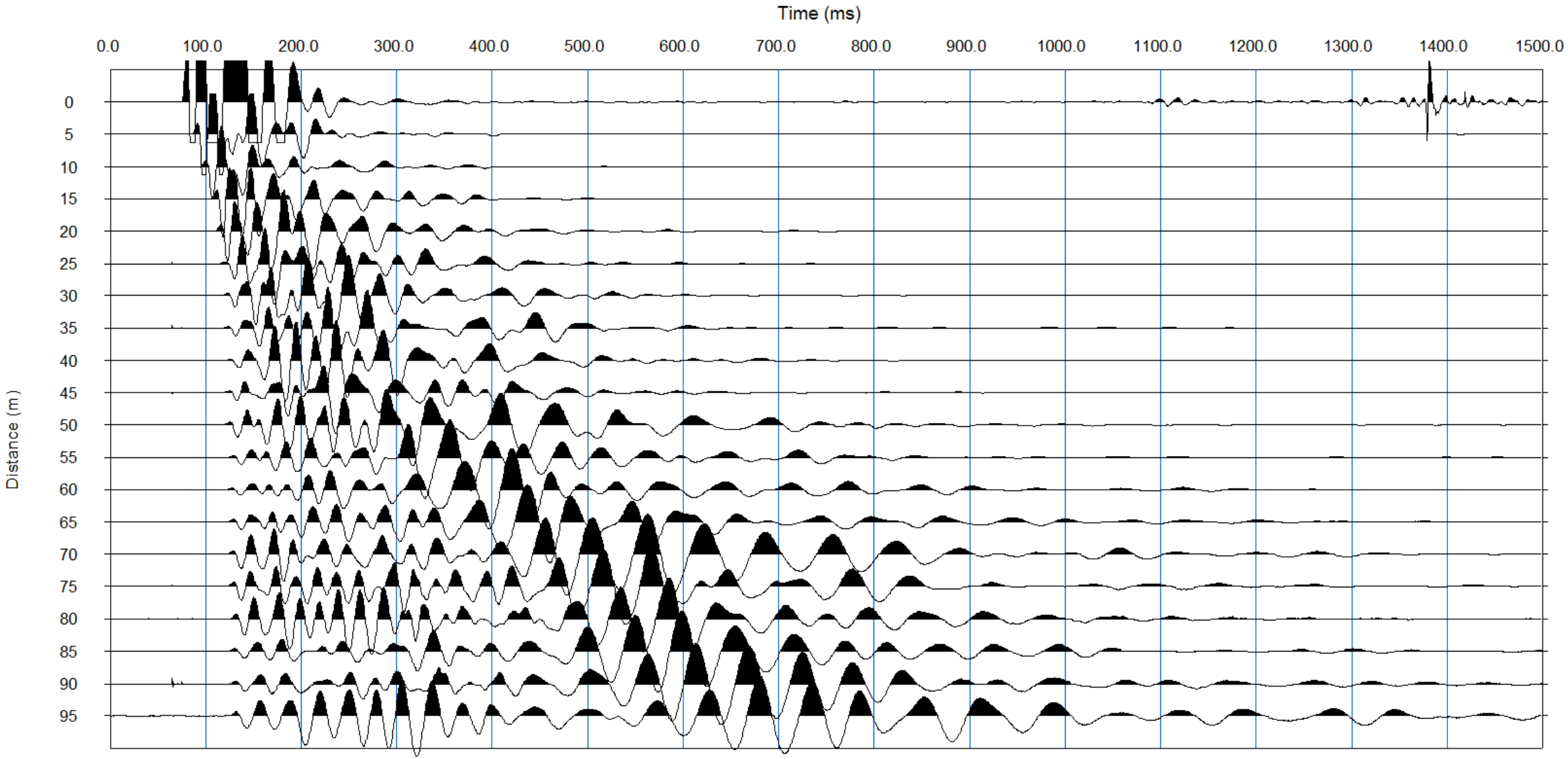
Lowland





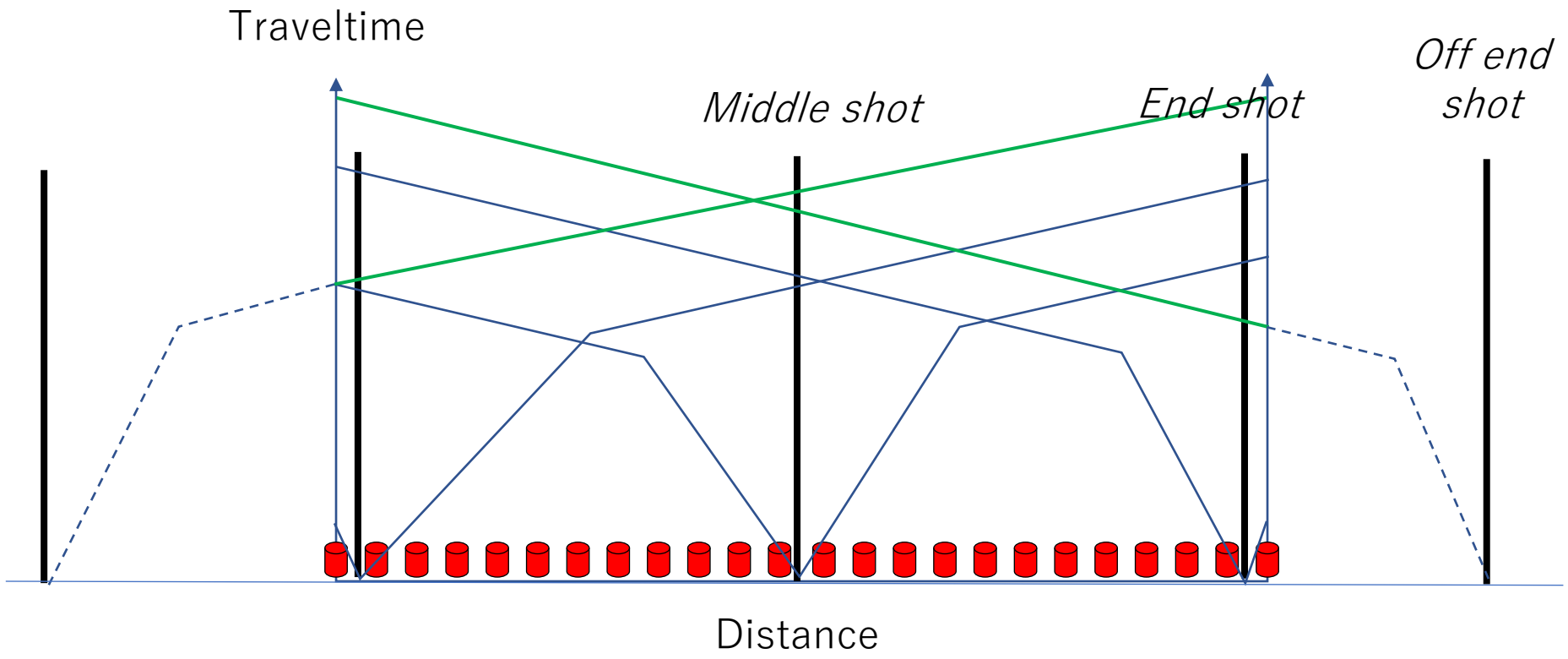


Typical refraction record



Refraction data acquisition for long survey line

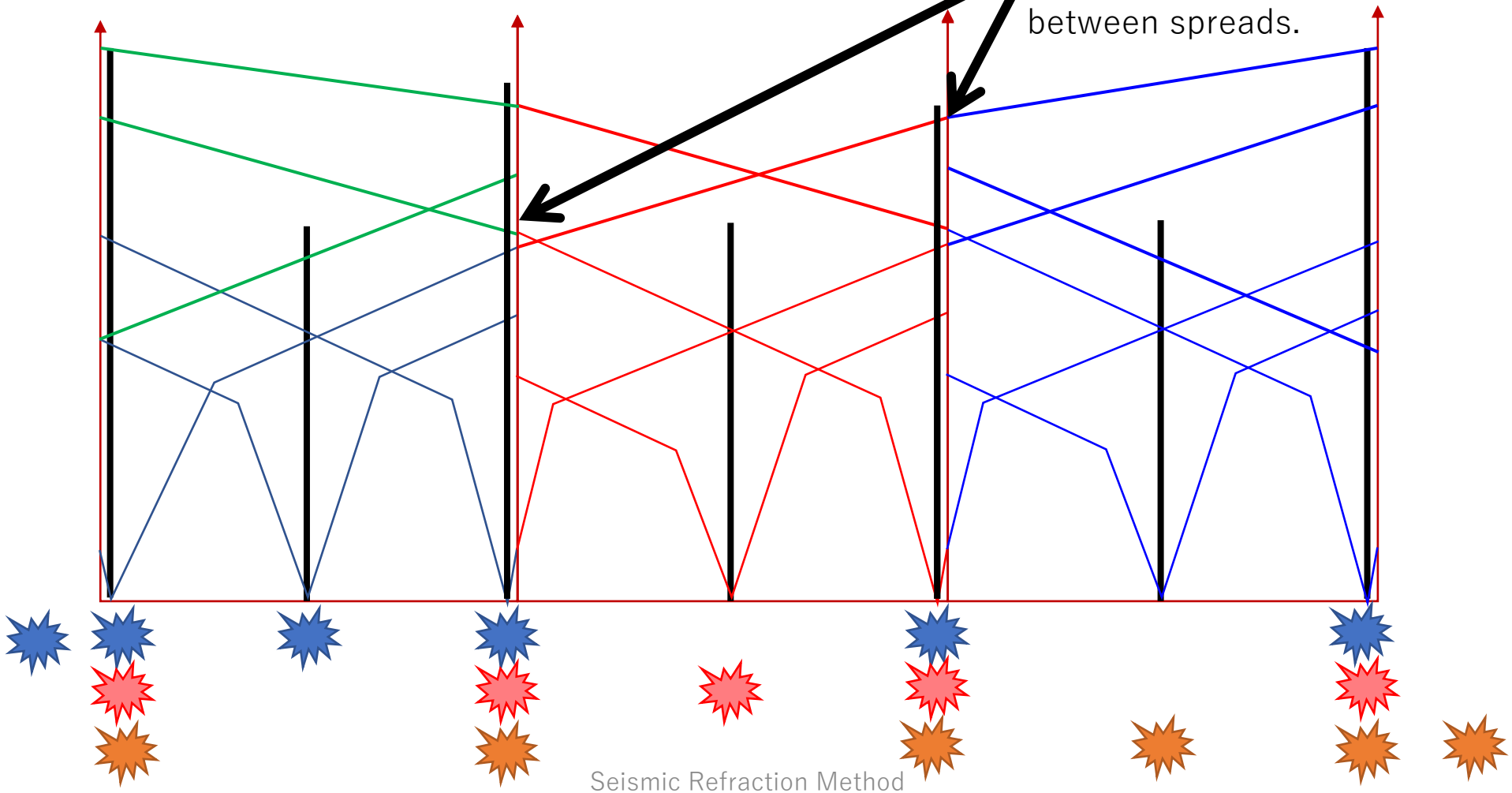
Traveltime curve for one spread (12 to 48 geophones)



Refraction data acquisition for long survey line

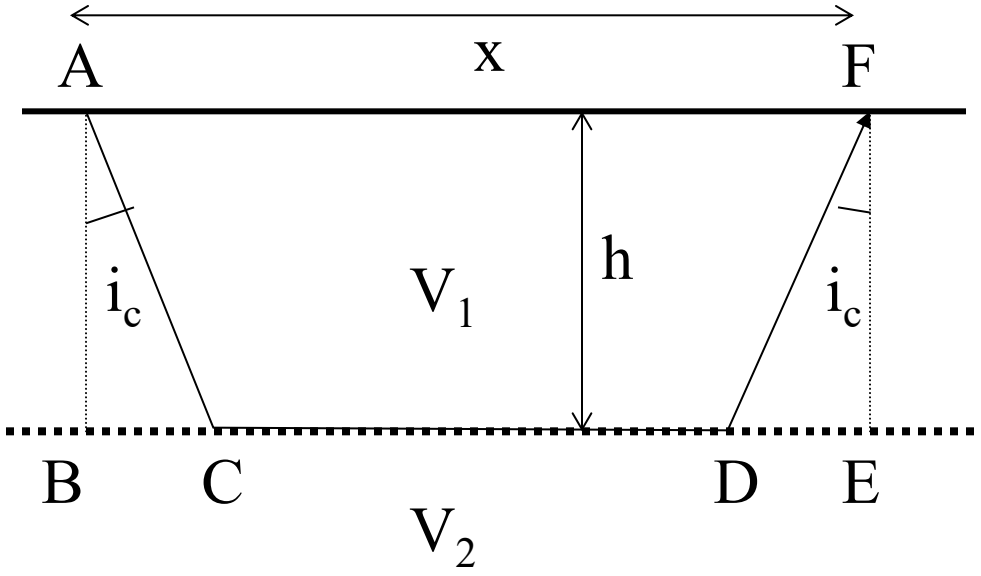
Example of a survey line consists of 3 spreads

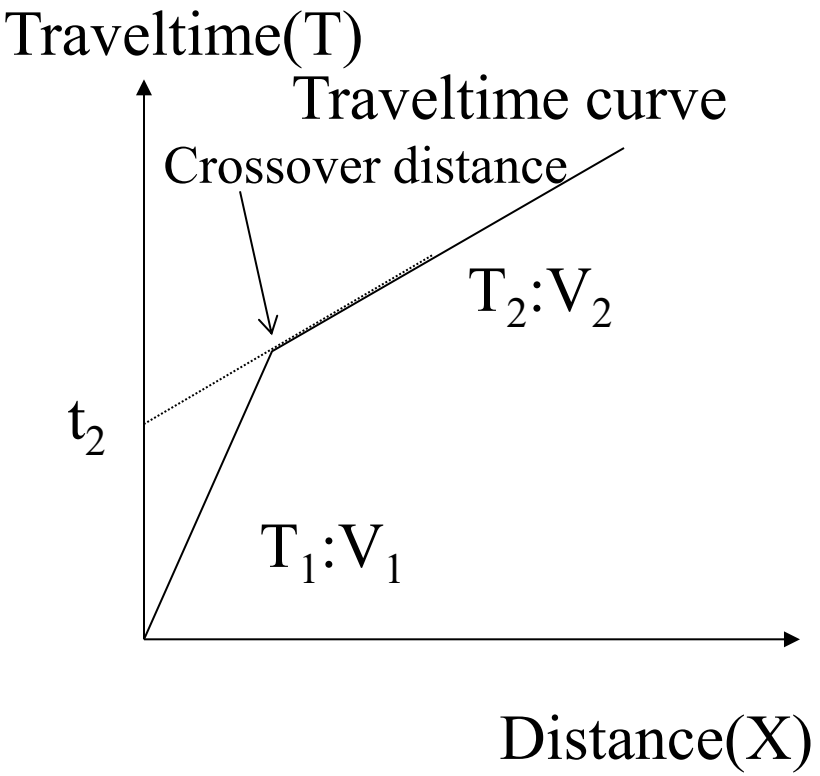
It is highly recommended to overlap 1 or 2 receivers(s) between spreads.



Fundamentals of seismic refraction

Horizontal two-layer model



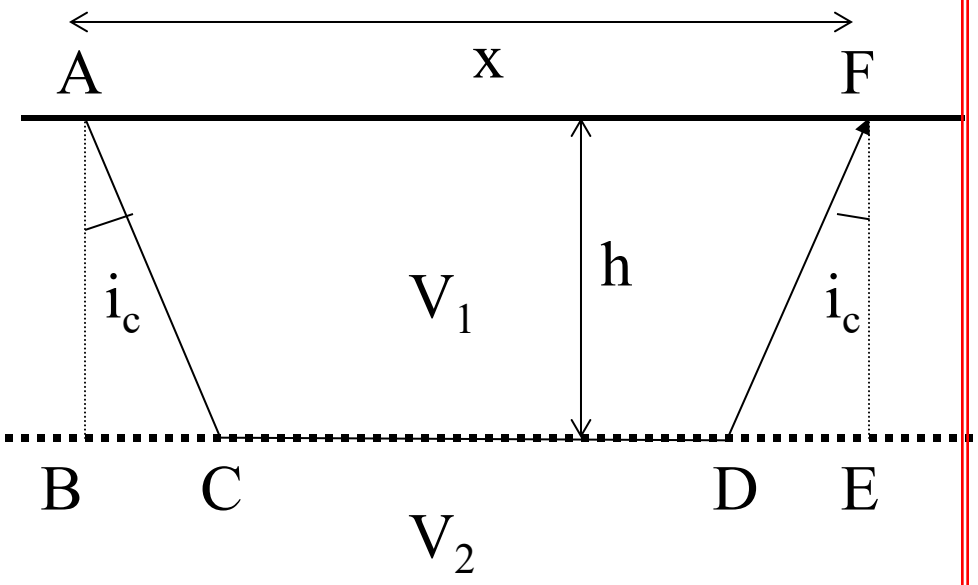


t_2=Intercept time

$$T_1 = \frac{X}{V_1} \qquad T_2 = ?$$

Fundamentals of seismic refraction

Horizontal two-layer model



$$T_2 = \frac{\overline{AC}}{V_1} + \frac{\overline{CD}}{V_2} + \frac{\overline{DF}}{V_1}$$

$$T_2 = \frac{2h\cos i_c}{V_1} + \frac{x}{V_2}$$

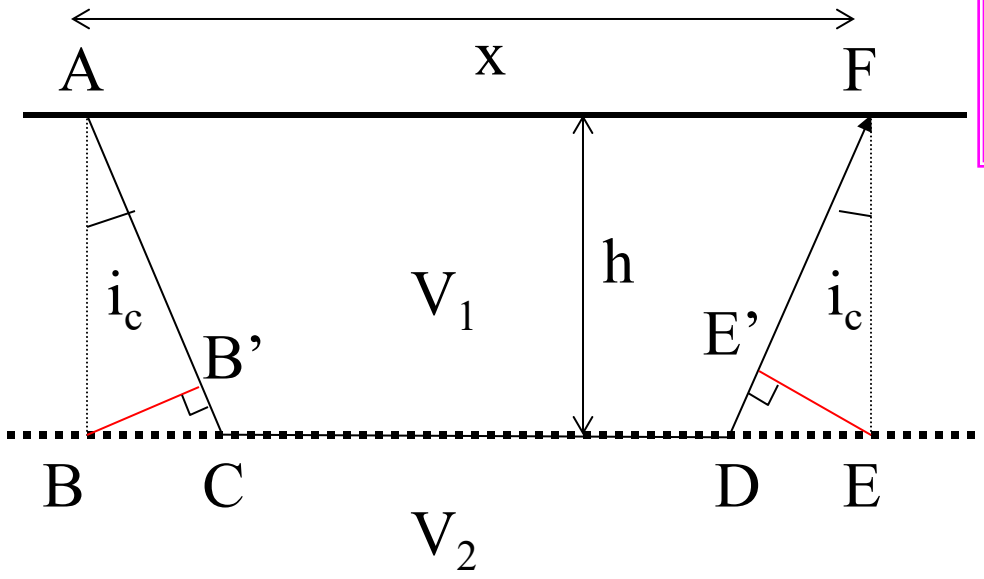
Intercept time

Snell's law

$$\sin i_c = \frac{V_1}{V_2}$$

Fundamentals of seismic refraction

Horizontal two-layer model



$$T_2 = \frac{\overline{AC}}{V_1} + \frac{\overline{CD}}{V_2} + \frac{\overline{DF}}{V_1}$$

$$T_2 = \frac{\overline{AB'}}{V_1} + \frac{\overline{B'C}}{V_1} + \frac{\overline{CD}}{V_2} + \frac{\overline{DE'}}{V_1} + \frac{\overline{E'F}}{V_1}$$

$$\overline{AB'} = h\cos i_c$$

$$\overline{E'F} = h\cos i_c$$

$$\overline{B'C} = \overline{BC}\sin i_c$$

Snell's law

$$\sin i_c = \frac{V_1}{V_2}$$

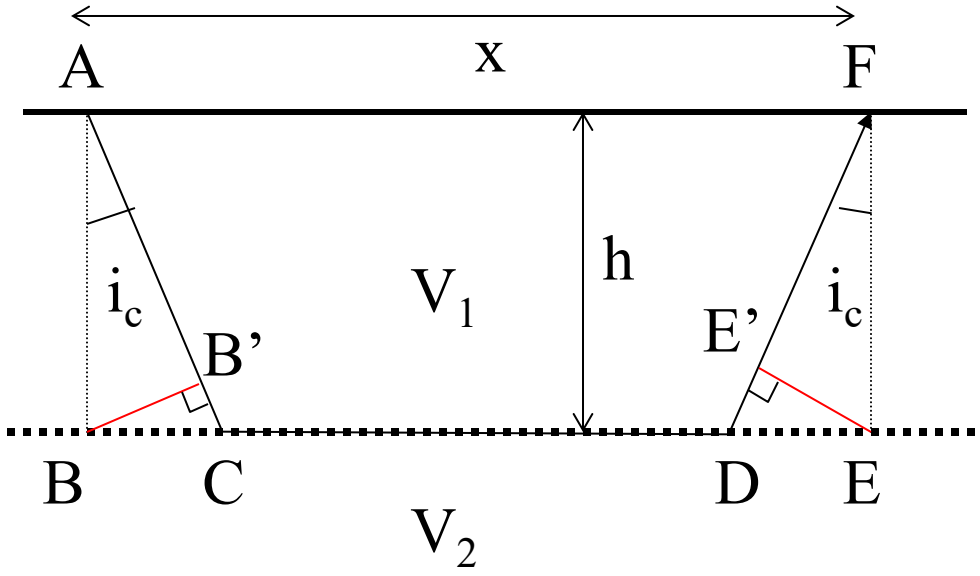
$$\frac{\overline{B'C}}{V_1} = \frac{\overline{BC}\sin i_c}{V_1} = \frac{\overline{BC}}{V_2}$$

$$\frac{\overline{DE'}}{V_1} = \frac{\overline{DE}\sin i_c}{V_1} = \frac{\overline{DE}}{V_2}$$

Fundamentals of seismic refraction

Horizontal two-layer model

$$T_2 = \frac{\overline{AB'}}{V_1} + \frac{\overline{B'C}}{V_1} + \frac{\overline{CD}}{V_2} + \frac{\overline{DE'}}{V_1} + \frac{\overline{E'F}}{V_1}$$



$$\overline{AB'} = h\cos i_c$$
$$\overline{E'F} = h\cos i_c$$

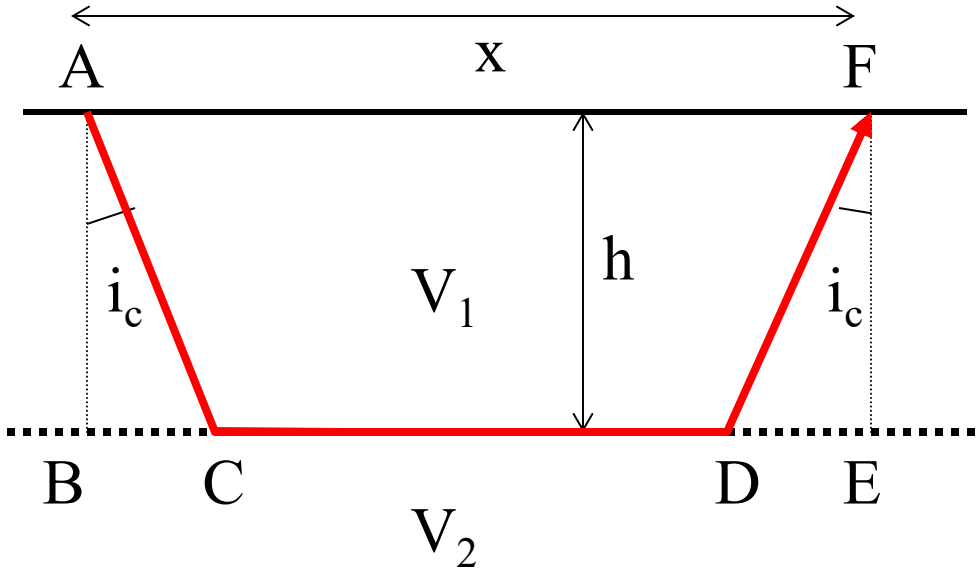
$$\frac{\overline{B'C}}{V_1} = \frac{\overline{BC}\sin i_c}{V_1} = \frac{\overline{BC}}{V_2}$$
$$\frac{\overline{DE'}}{V_1} = \frac{\overline{DE}\sin i_c}{V_1} = \frac{\overline{DE}}{V_2}$$

$$T_2 = \frac{h\cos i_c}{V_1} + \frac{\overline{BC}}{V_2} + \frac{\overline{CD}}{V_2} + \frac{\overline{DE}}{V_2} + \frac{h\cos i_c}{V_1}$$

$$T_2 = \frac{2h\cos i_c}{V_1} + \frac{x}{V_2}$$

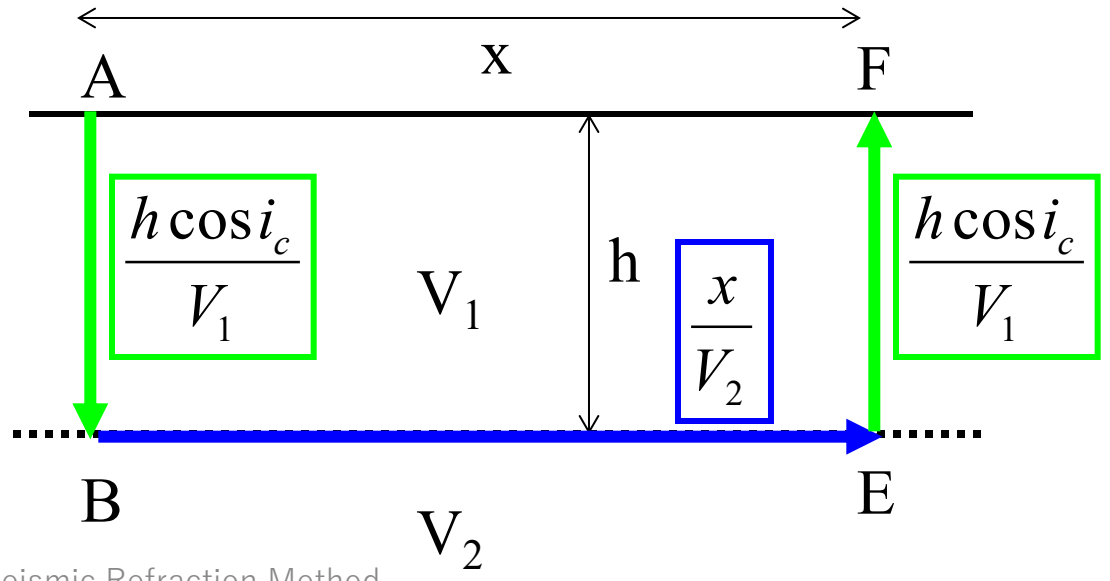
Fundamentals of seismic refraction

Physical expression



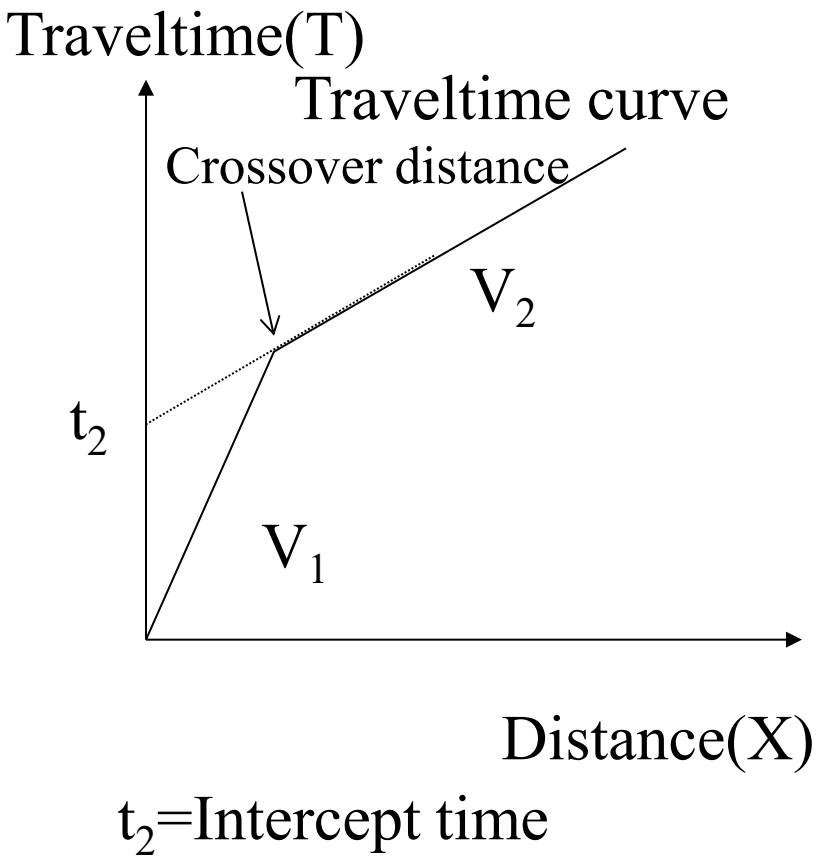
Mathematical expression

$$T_2 = \frac{2h\cos i_c}{V_1} + \frac{x}{V_2}$$



Fundamentals of seismic refraction

Horizontal two-layer model



t_2=Intercept time

$$T_2 = \frac{2h\cos i_c}{V_1} + \frac{x}{V_2}$$

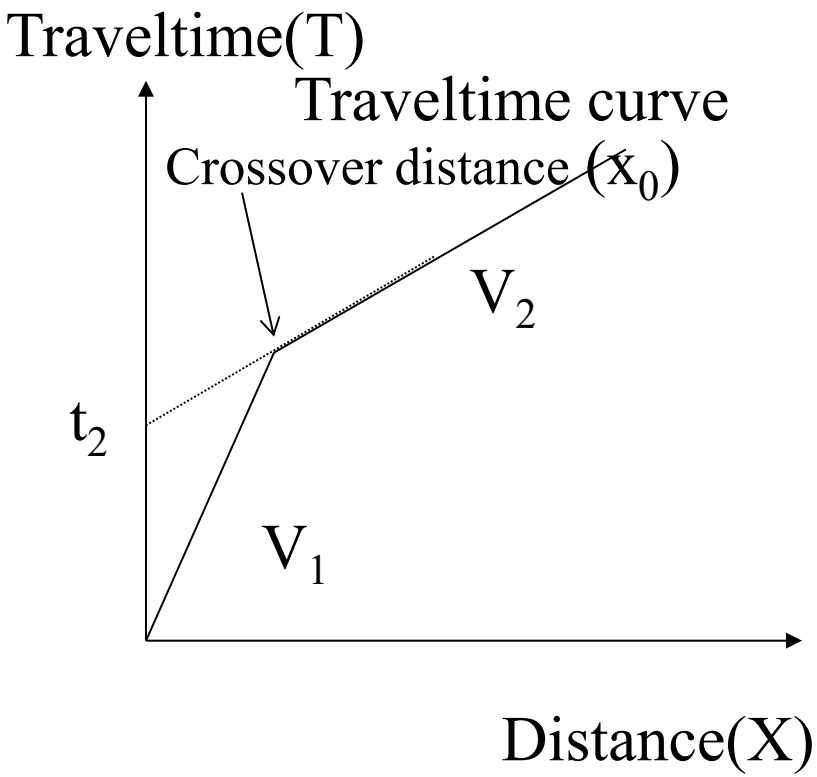
$$\tau_2 = \frac{t_2}{2} = \frac{h\cos i_c}{V_1}$$

τ_2 = Delay term

$$h = \frac{V_1 t_2}{2\cos i_c} = \frac{V_1 \tau_2}{\cos i_c} = \text{Depth}$$

Fundamentals of seismic refraction

Horizontal two-layer model



t_2=Intercept time

Crossover distance : x_0

$$T_1 = T_2$$

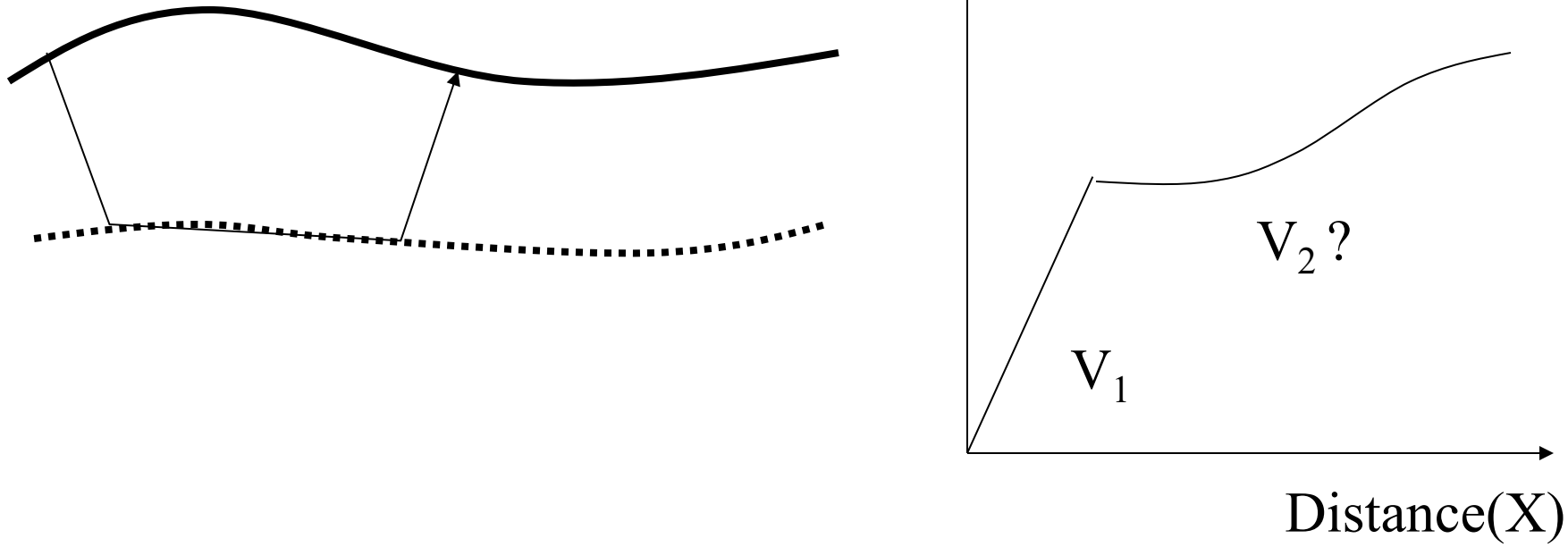
$$T_2 = \frac{2h\cos i_c}{V_1} + \frac{x_0}{V_2} = \frac{x_0}{V_1} = T_1$$

$$x_0 = 2h\sqrt{\frac{V_2 + V_1}{V_2 - V_1}} = \text{Crossover distance}$$

$$h = \frac{x_0}{2}\sqrt{\frac{V_2 - V_1}{V_2 + V_1}} = \text{Depth}$$

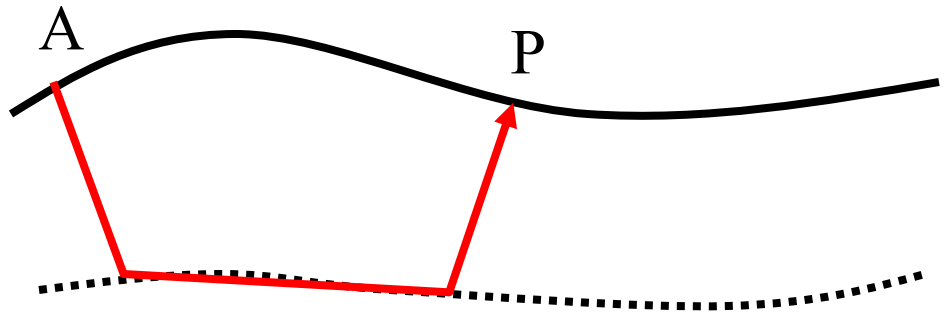
Fundamentals of seismic refraction

Irregular layer boundary

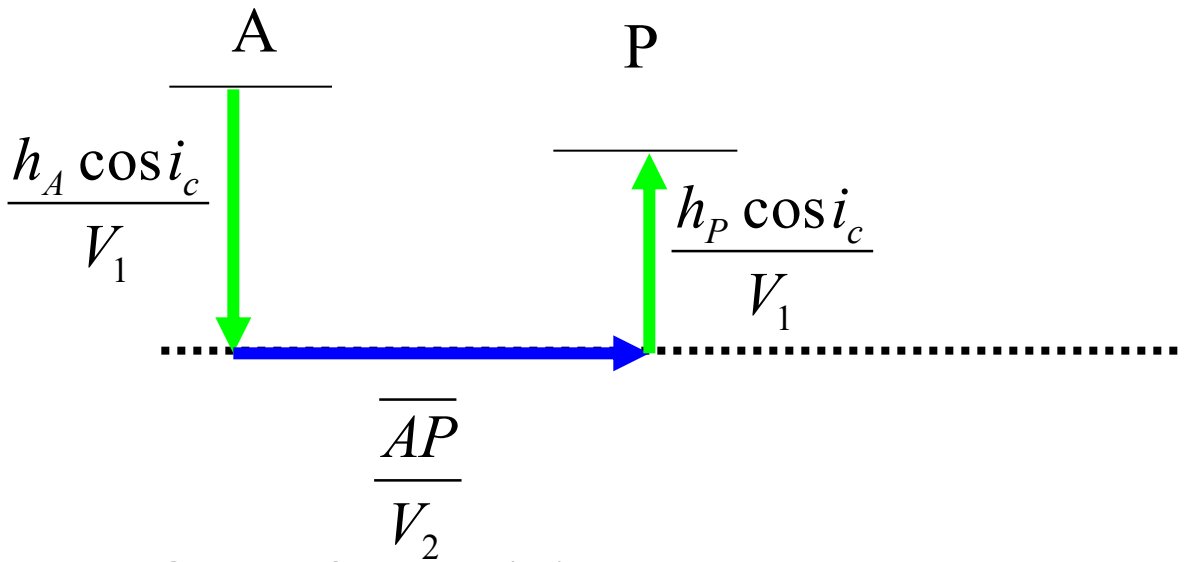


Fundamentals of seismic refraction

Physical expression

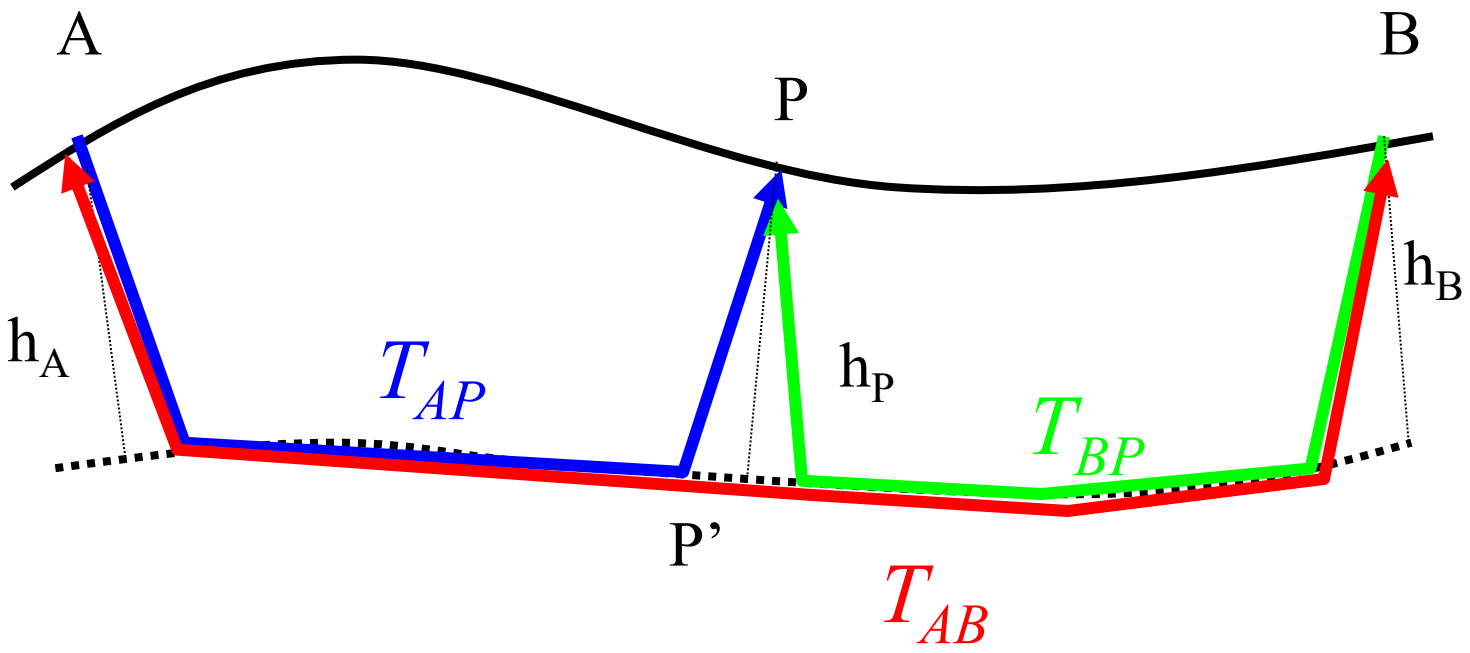


Mathematical expression



Fundamentals of seismic refraction

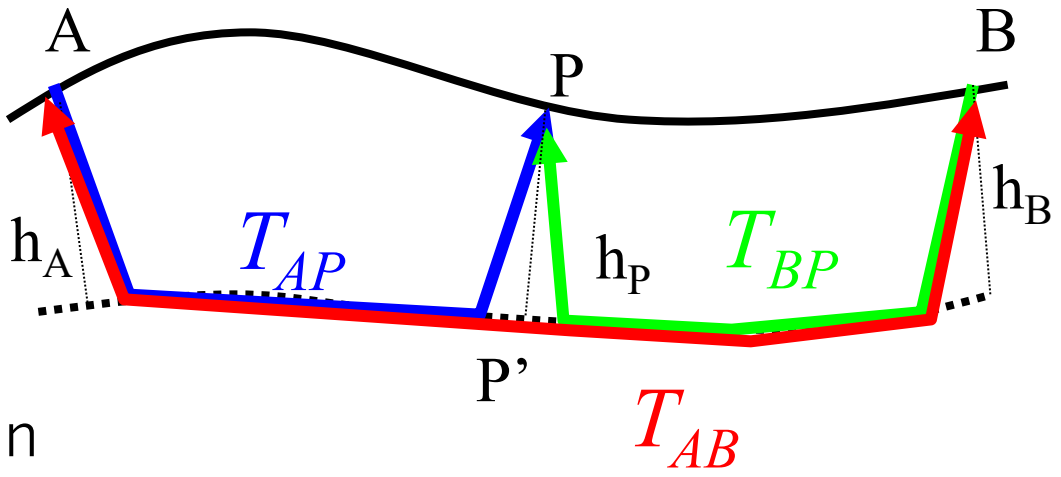
Reciprocal method !
Use three traveltimes!



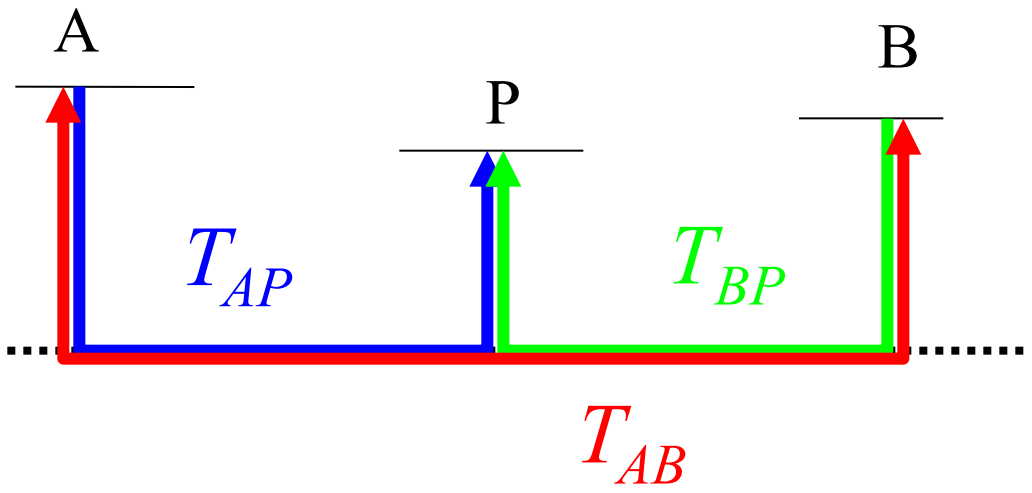
Reciprocal traveltime

Fundamentals of seismic refraction

Physical expression

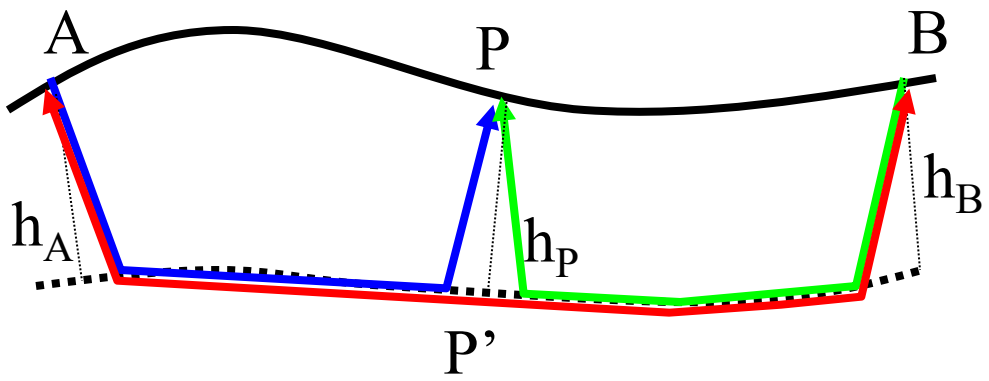


Mathematical expression



Fundamentals of seismic refraction

Reciprocal method !
Use three traveltimes!



$$T_{AP} \cong \frac{h_A \cos i_c}{V_1} + \frac{\overline{AP}}{V_2} + \frac{h_P \cos i_c}{V_1}$$

$$T_{BP} \cong \frac{h_B \cos i_c}{V_1} + \frac{\overline{BP}}{V_2} + \frac{h_P \cos i_c}{V_1}$$

$$T_{AB} \cong \frac{h_A \cos i_c}{V_1} + \frac{\overline{AB}}{V_2} + \frac{h_B \cos i_c}{V_1}$$

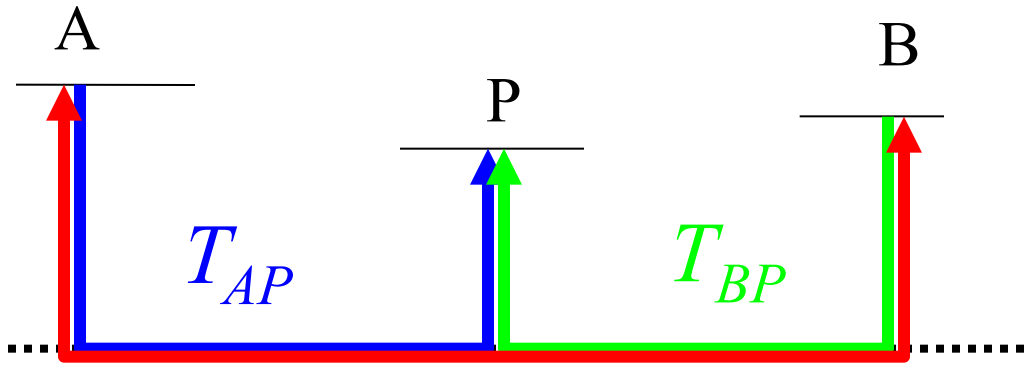
$$\downarrow$$

$$t_0 = T_{AP} + T_{BP} - T_{AB} = \frac{2h_P \cos i_c}{V_1}$$

t_0 is the time related to the 1st layer thickness at the point of P.
($t_0/2$ is called as "Delay time(time-term)")

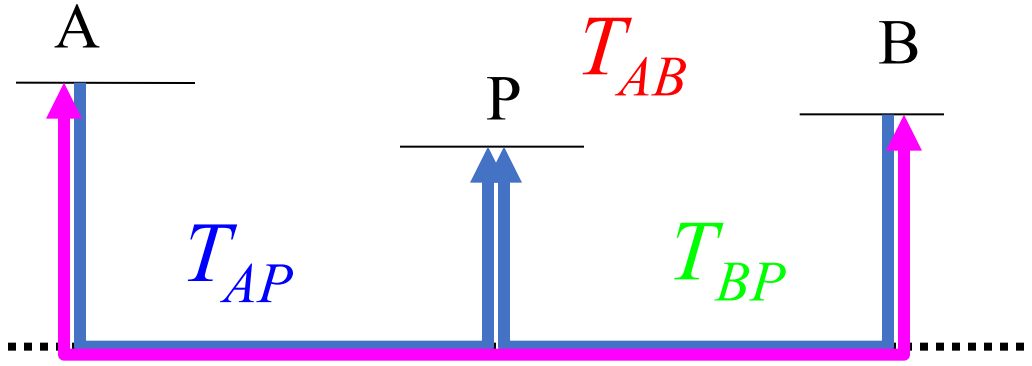
Fundamentals of seismic refraction

Three ray paths in mathematical expression

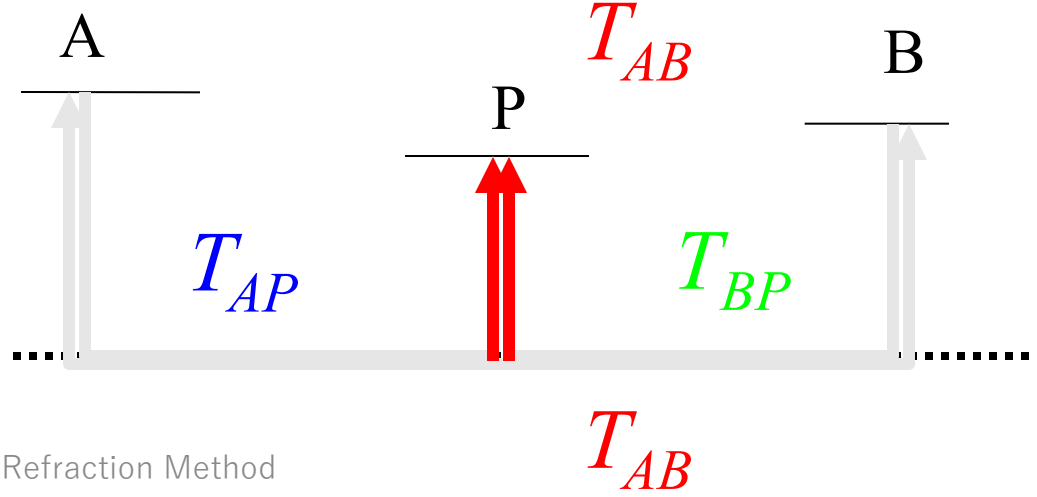


Calculating t_0

$$t_0 = \boxed{T_{AP} + T_{BP}} - \boxed{T_{AB}} = \frac{2h_P \cos i_c}{V_1}$$



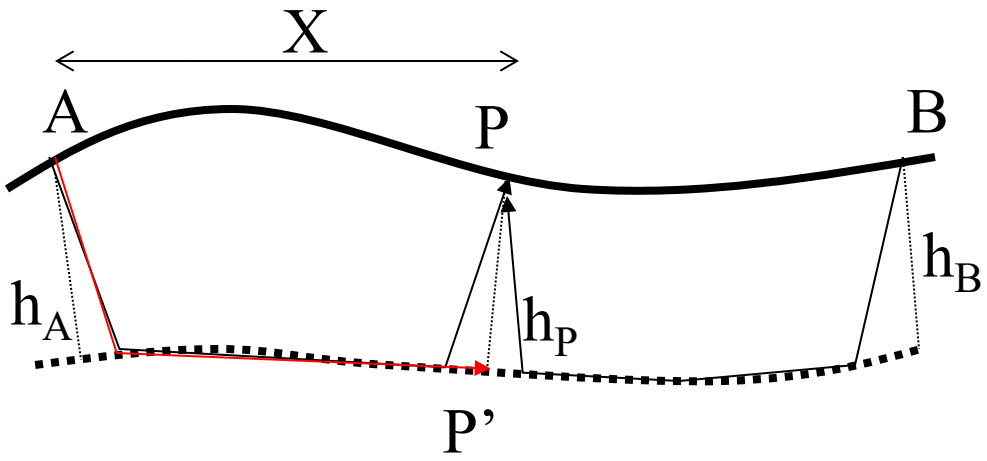
$$t_0 = T_{AP} + T_{BP} - T_{AB} = \boxed{\frac{2h_P \cos i_c}{V_1}}$$



Fundamentals of seismic refraction

Irregular boundary

Introduction of T'(Reduced traveltime)



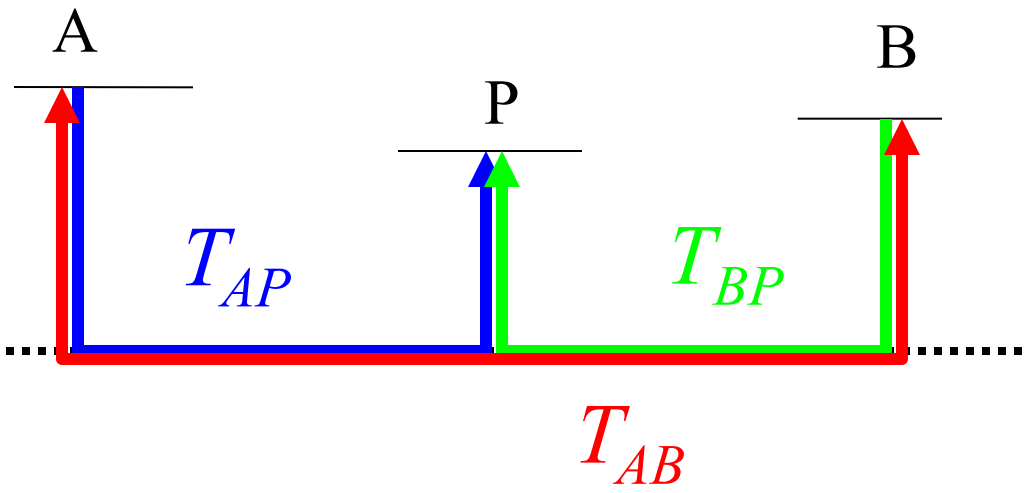
Delay time at A

$$T'_{AP} = T_{AP'} = T_{AP} - \frac{t_0}{2} = T_{AP} - \frac{(T_{AP} + T_{BP} - T_{AB})}{2} \cong \frac{h_A \cos i_c}{V_1} + \frac{x}{V_2}$$

The slope of T'_{AP} is equal to the velocity of V_2 in spite of 1st layer thickness.

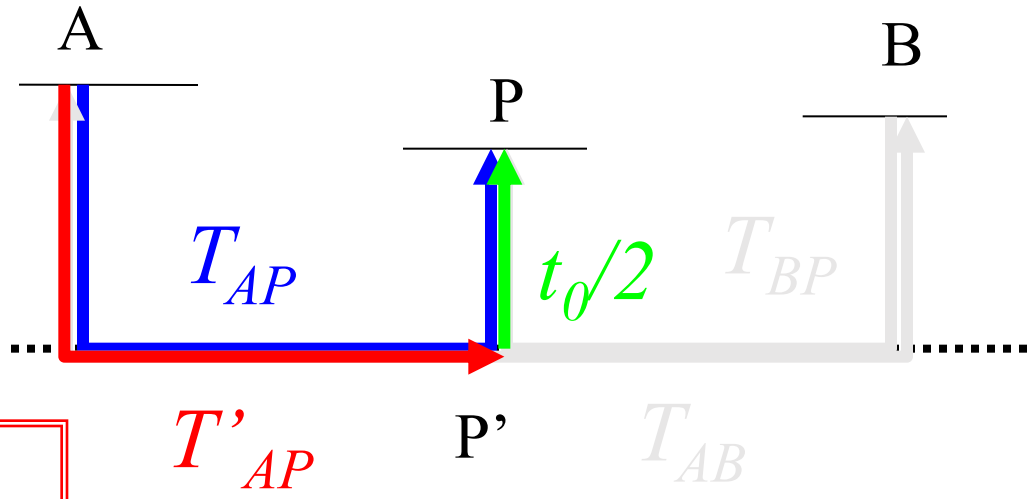
Fundamentals of seismic refraction

Reciprocal traveltime



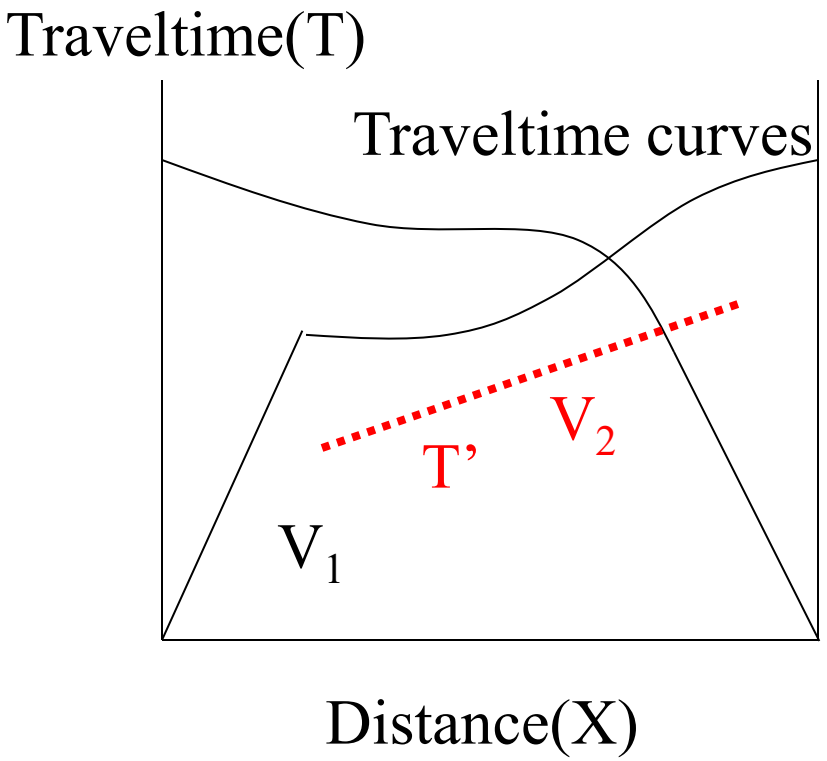
$$T'_{AP} = T_{AP} - \frac{t_0}{2}$$

Delay time : d_p



Fundamentals of seismic refraction

T’ (reduced traveltime) on traveltime curves

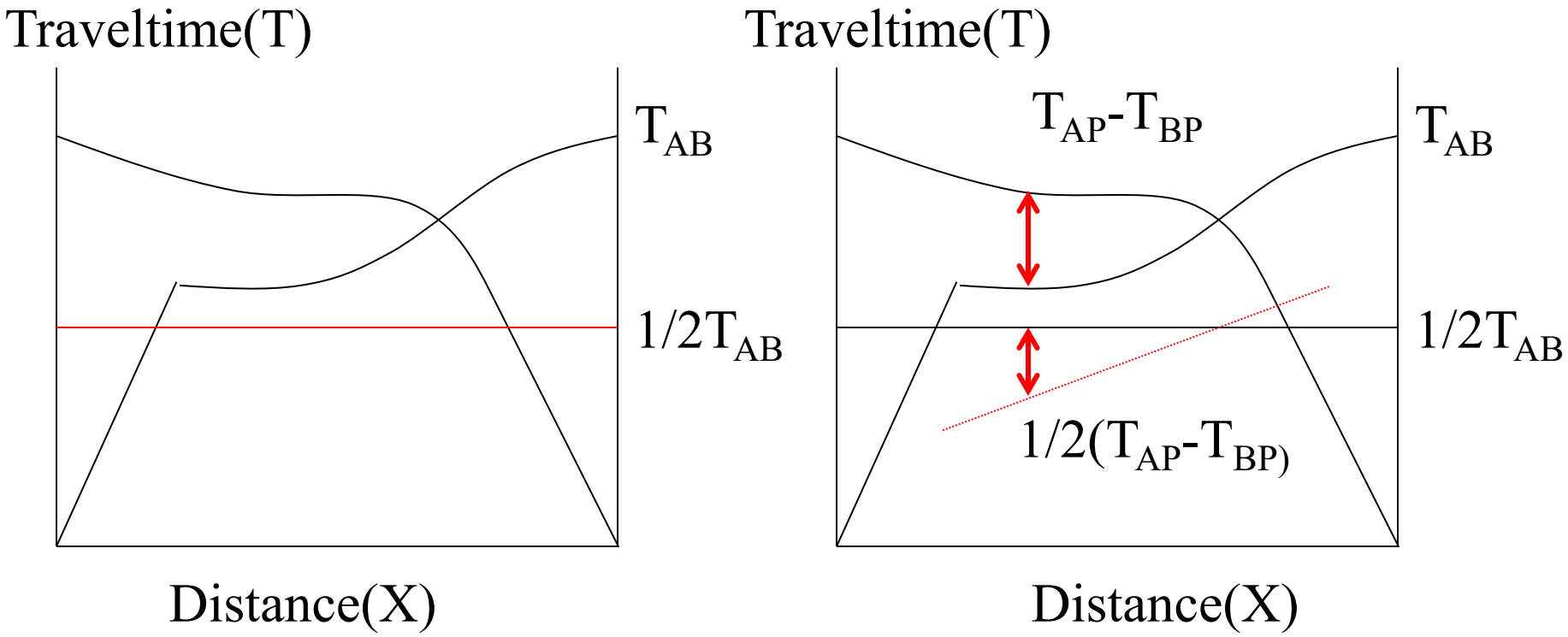


Fundamentals of seismic refraction

The calculation of T' by hand

$$T'_{AP} = T_{AP} - \frac{(T_{AP} + T_{BP} - T_{AB})}{2} = \frac{T_{AB}}{2} + \frac{(T_{AP} - T_{BP})}{2}$$

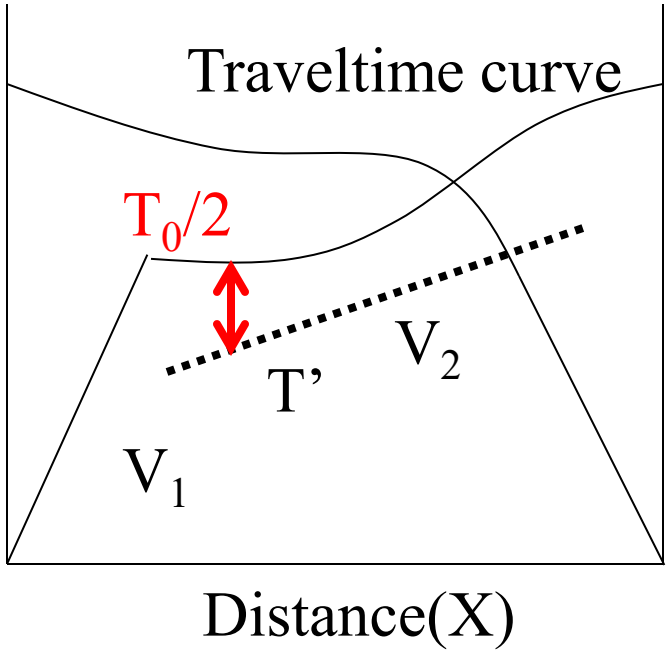
1 : Measure $1/2T_{AB}$ by a dividers and draw a line.
2 : Measure T_{AP}-T_{BP} by a dividers and divide it into halves,
3 : dot T'_{AP} below (or above) $1/2T_{AB}$.
4 : Fit a straight line on T'_{AP} dots. The slope of the straight line means a 2nd layer velocity.



Fundamentals of seismic refraction

Calculation of d_p (Delay time)

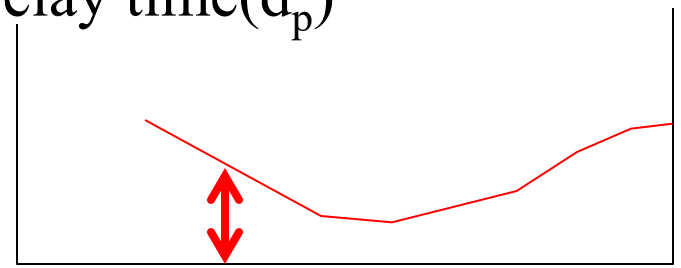
Traveltime(T)



$$d_p = \frac{t_0}{2} = T_{AP} - T'_{AP} = \frac{h_P \cos i_c}{V_1}$$

$$h_P = \frac{d_P V_1}{\cos i_c}$$

Delay time(d_p)



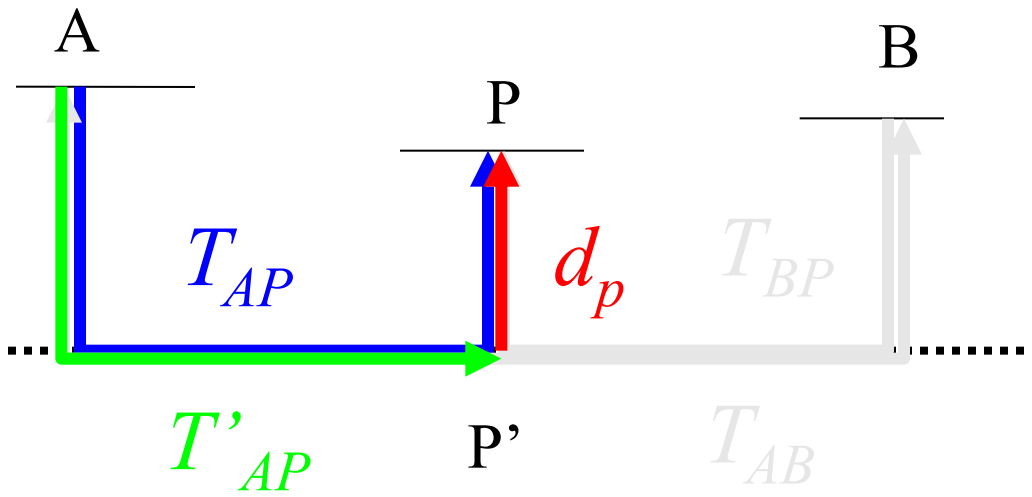
hp : thickness of 1st layer

1 :Measure a time from the straight line fitted on T' dots to each traveltimes.
2 : Plot it as delay time.

Fundamentals of seismic refraction

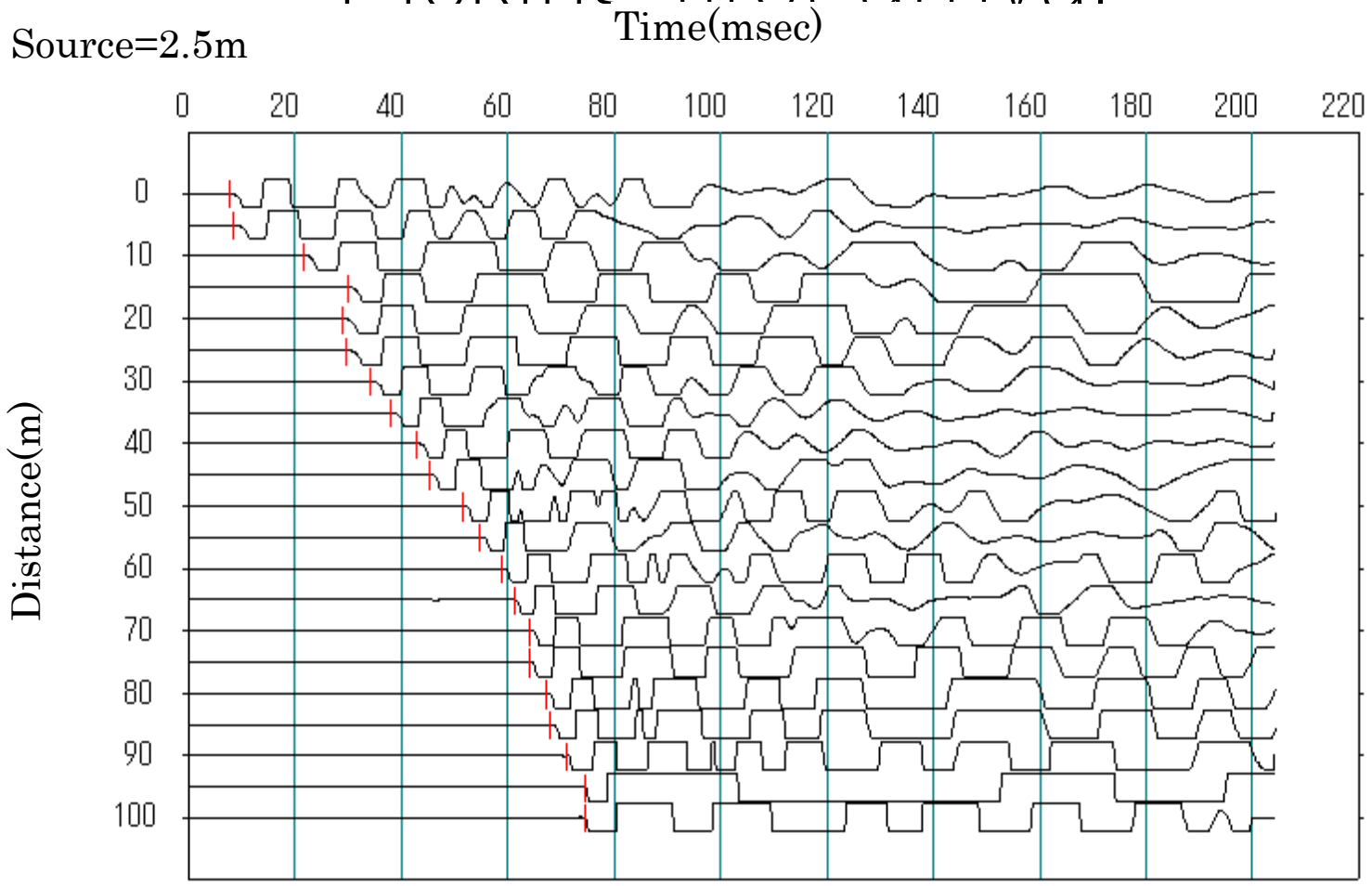
Mathematical meaning of delay time : d_p

$$d_p = T_{AP} - T'_{AP}$$



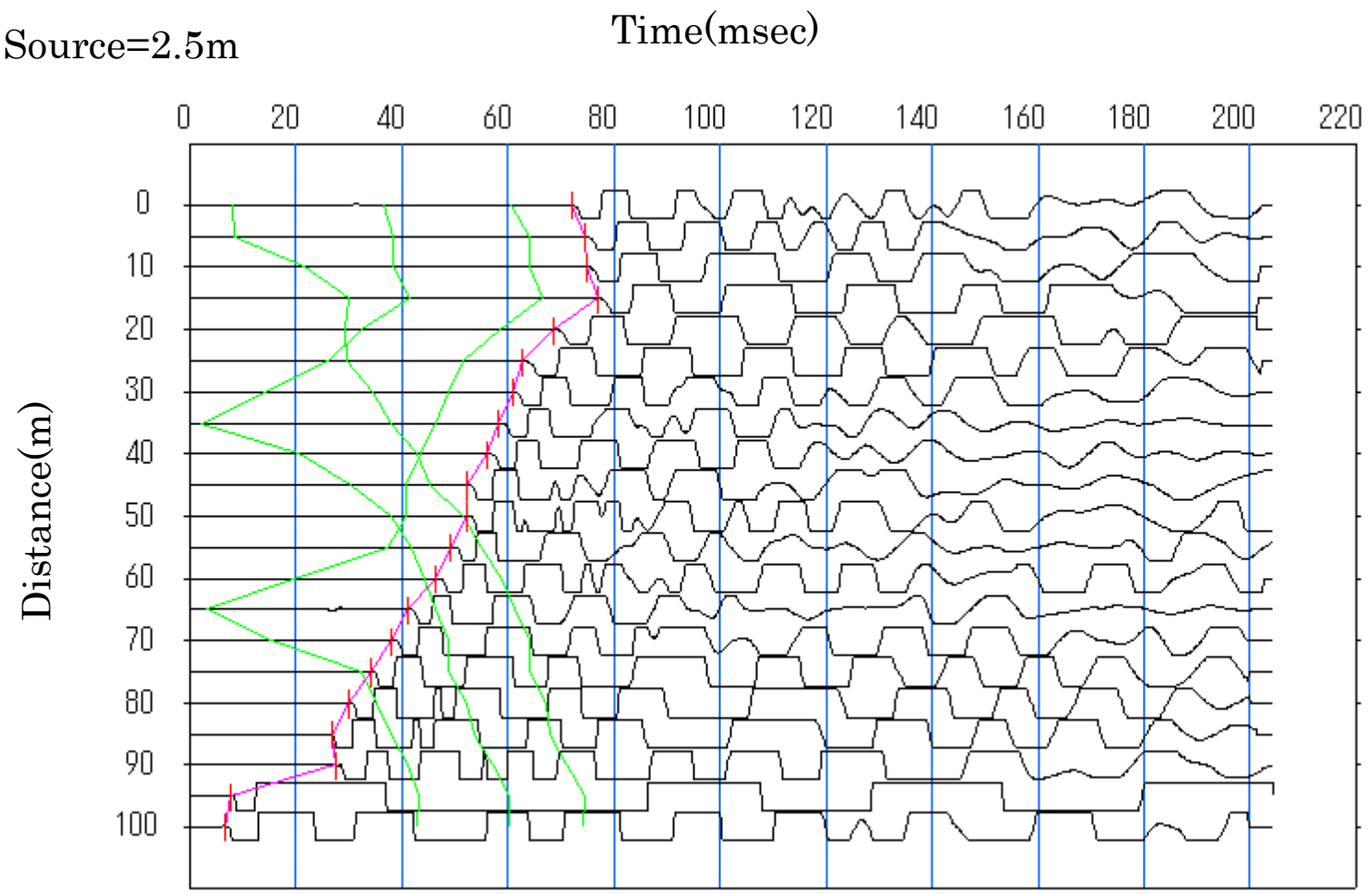
Reciprocal method analysis procedure

Picking first arrival

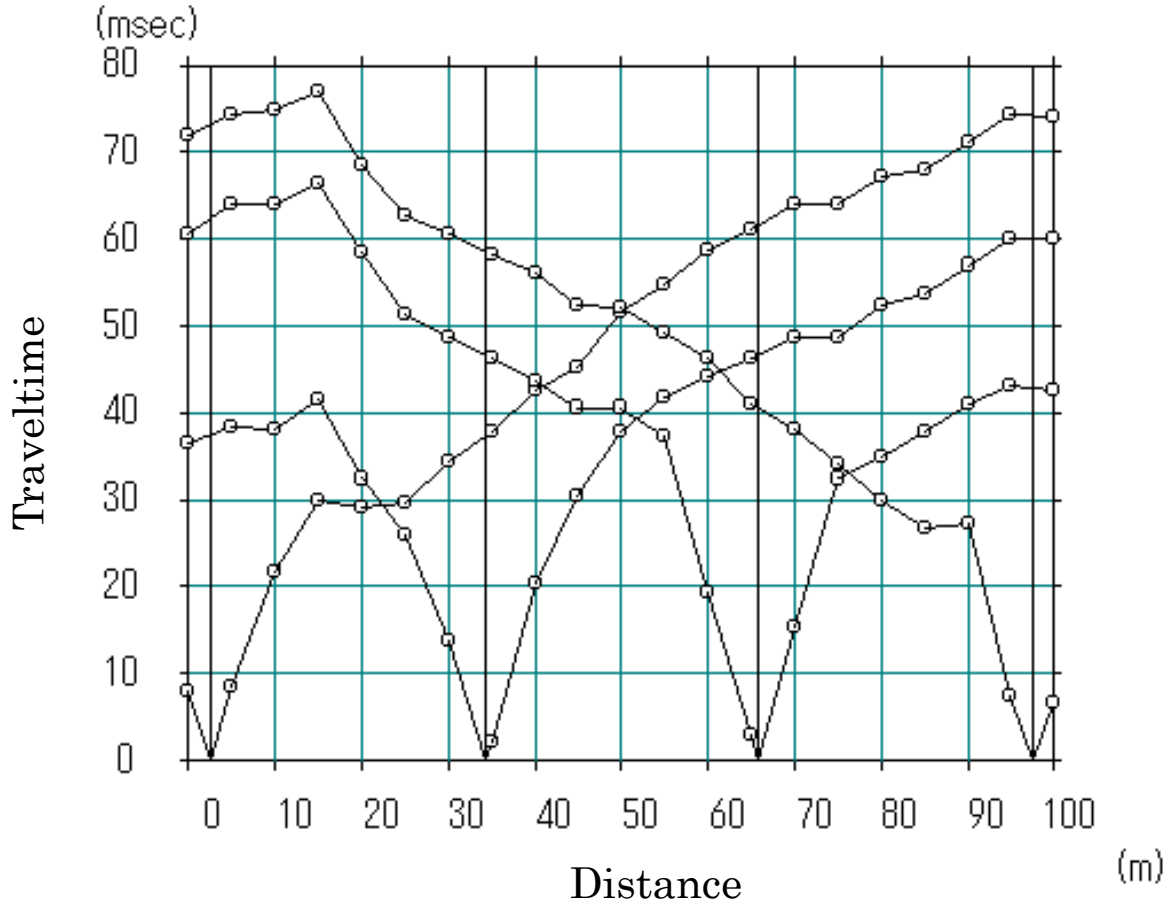


Picking first arrival

Source=2.5m



Traveltime curve (Time-distance curve)



Reciprocal method analysis procedure

Checking traveltime curves

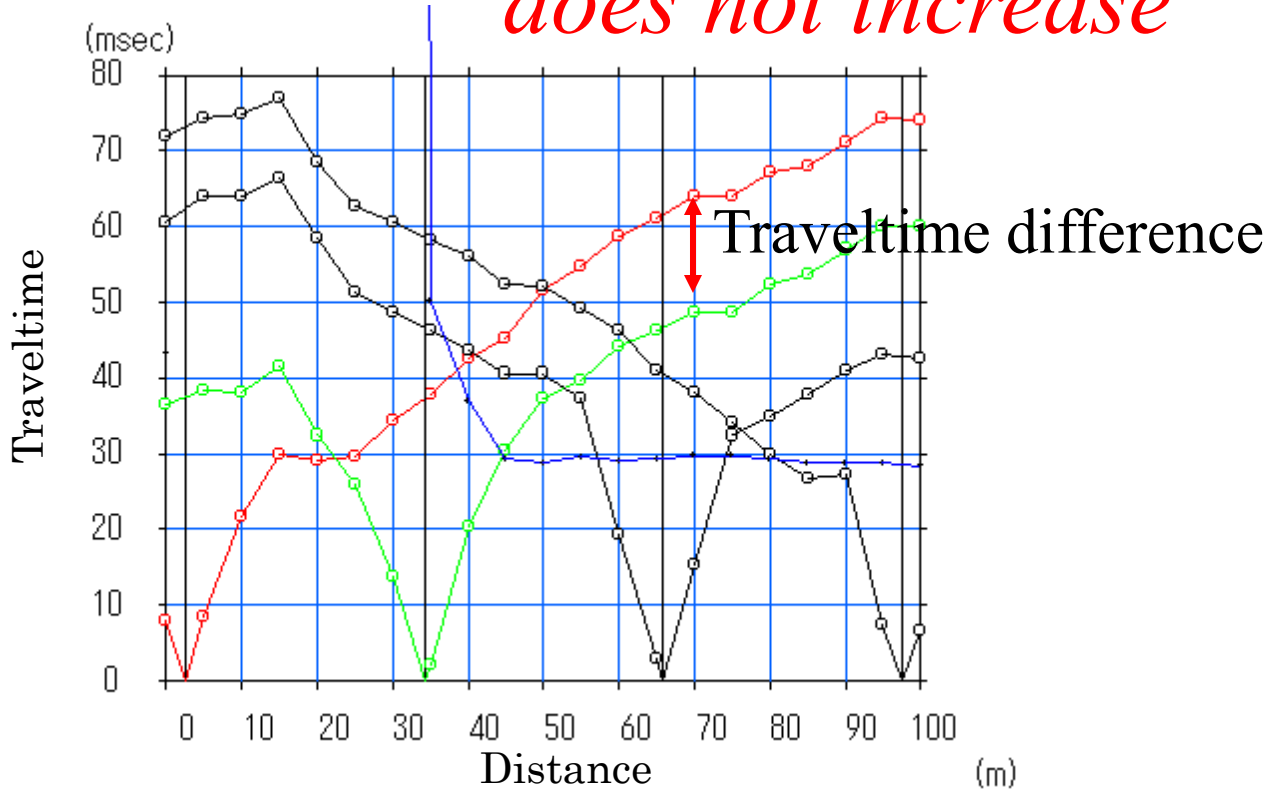
- Parallelism
- Reciprocal traveltime
- Crossover distance
- Intercept time

→ Completed traveltime curves

Checking traveltime curves

Parallelism

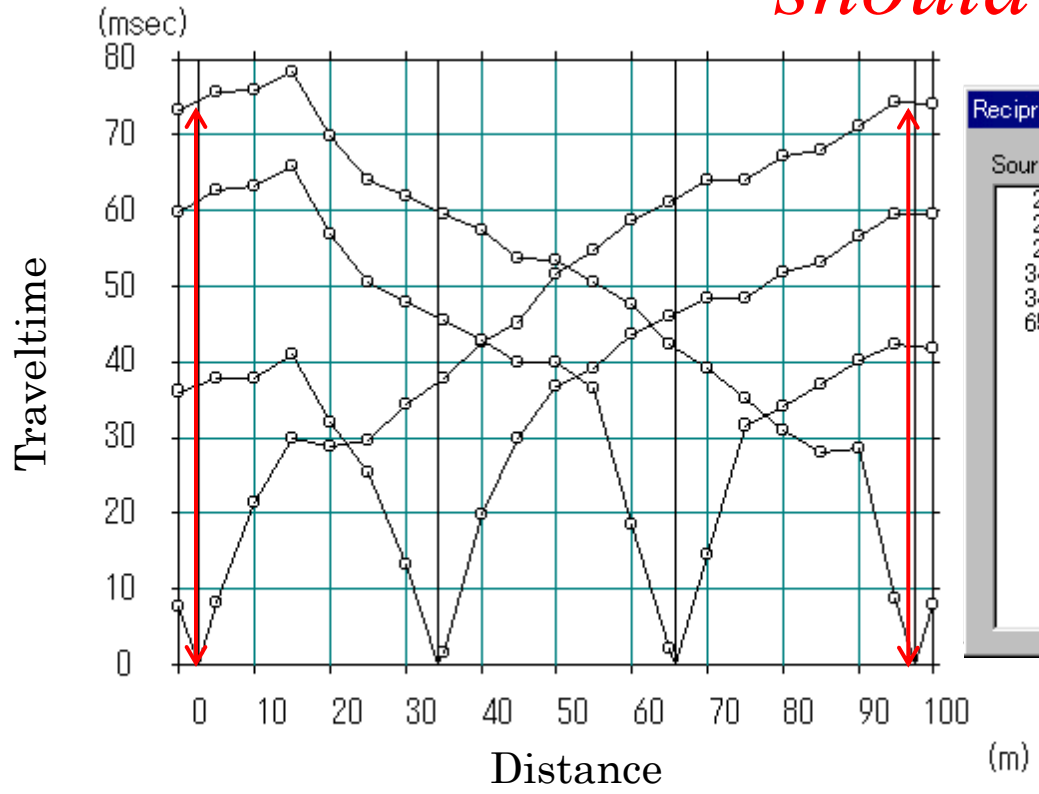
Traveltime difference does not increase

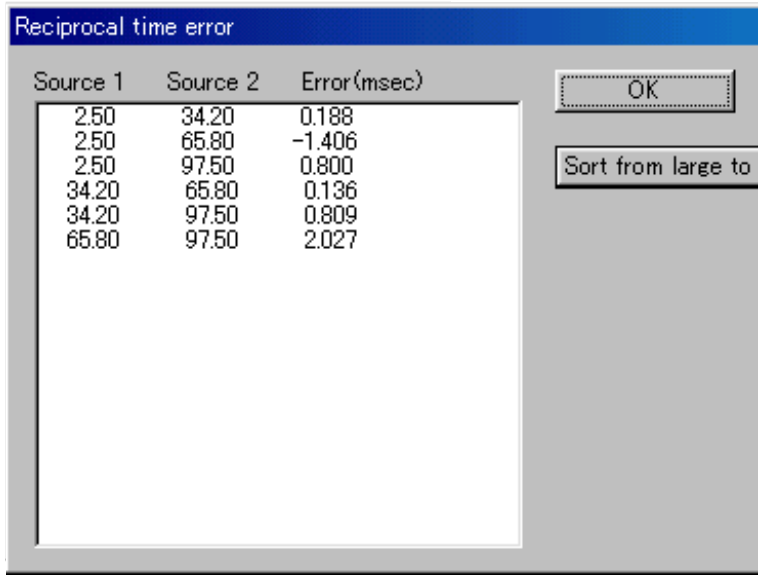


Checking traveltime curves

Reciprocal traveltime

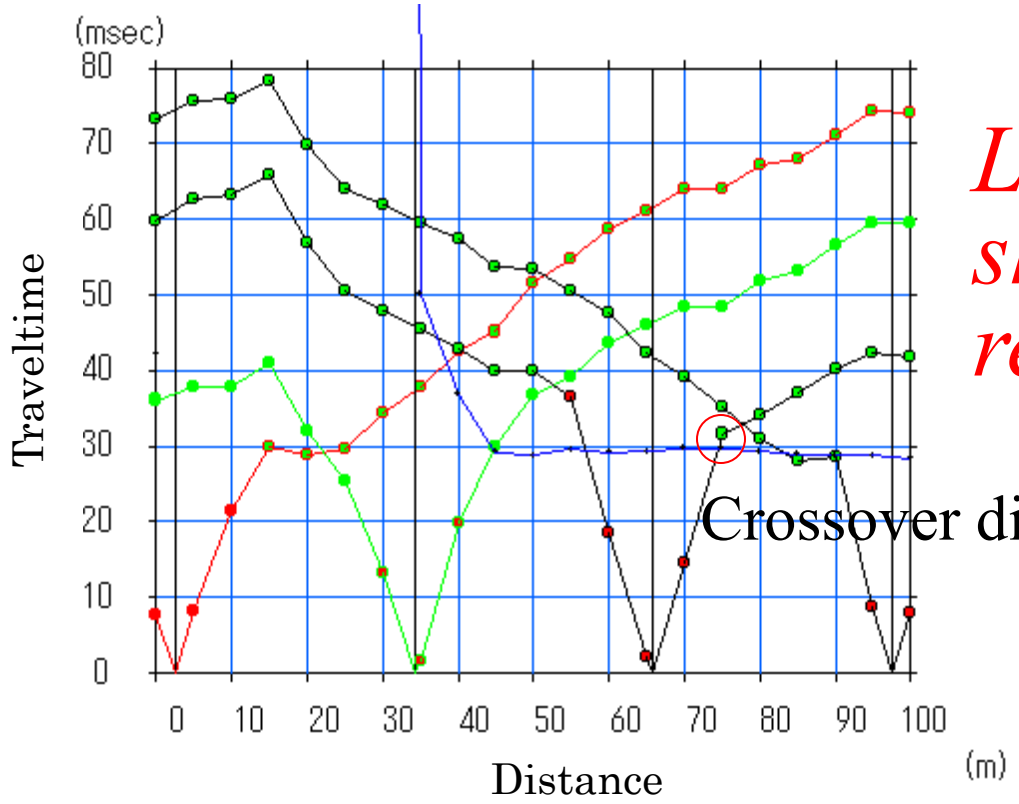
Reciprocal traveltime should be same





Checking traveltime curves

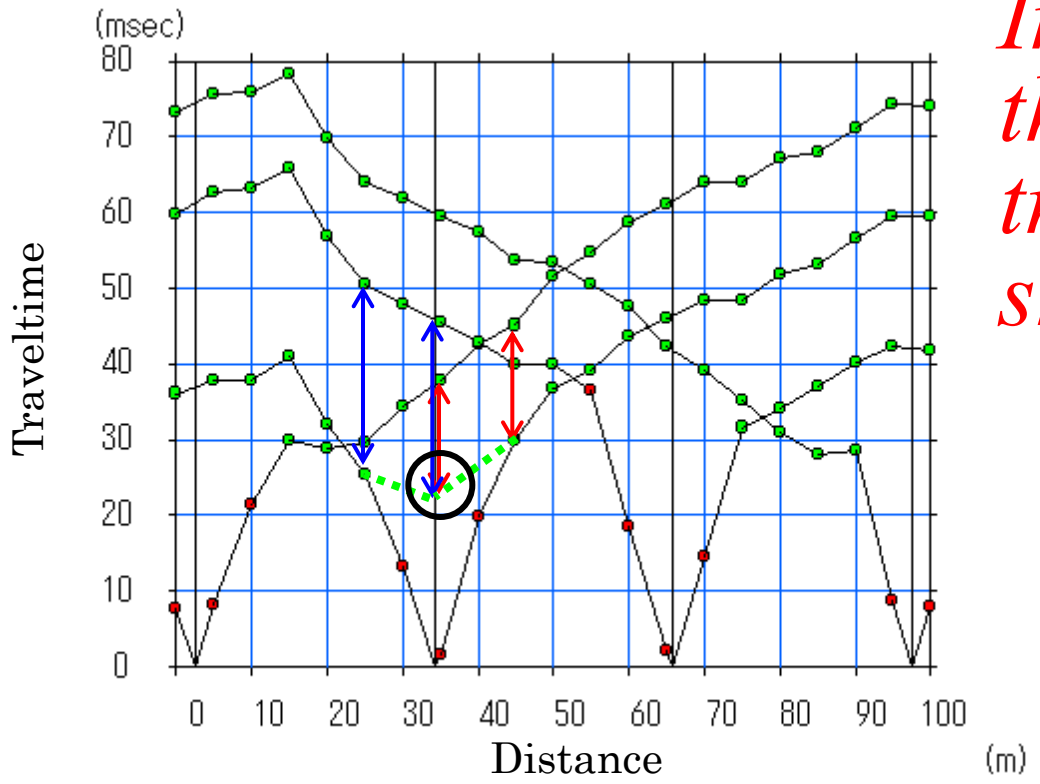
Crossover distance (layer assignment)



Layer assignment should be same in reciprocal traveltimes

Checking traveltime curves

Intercept time

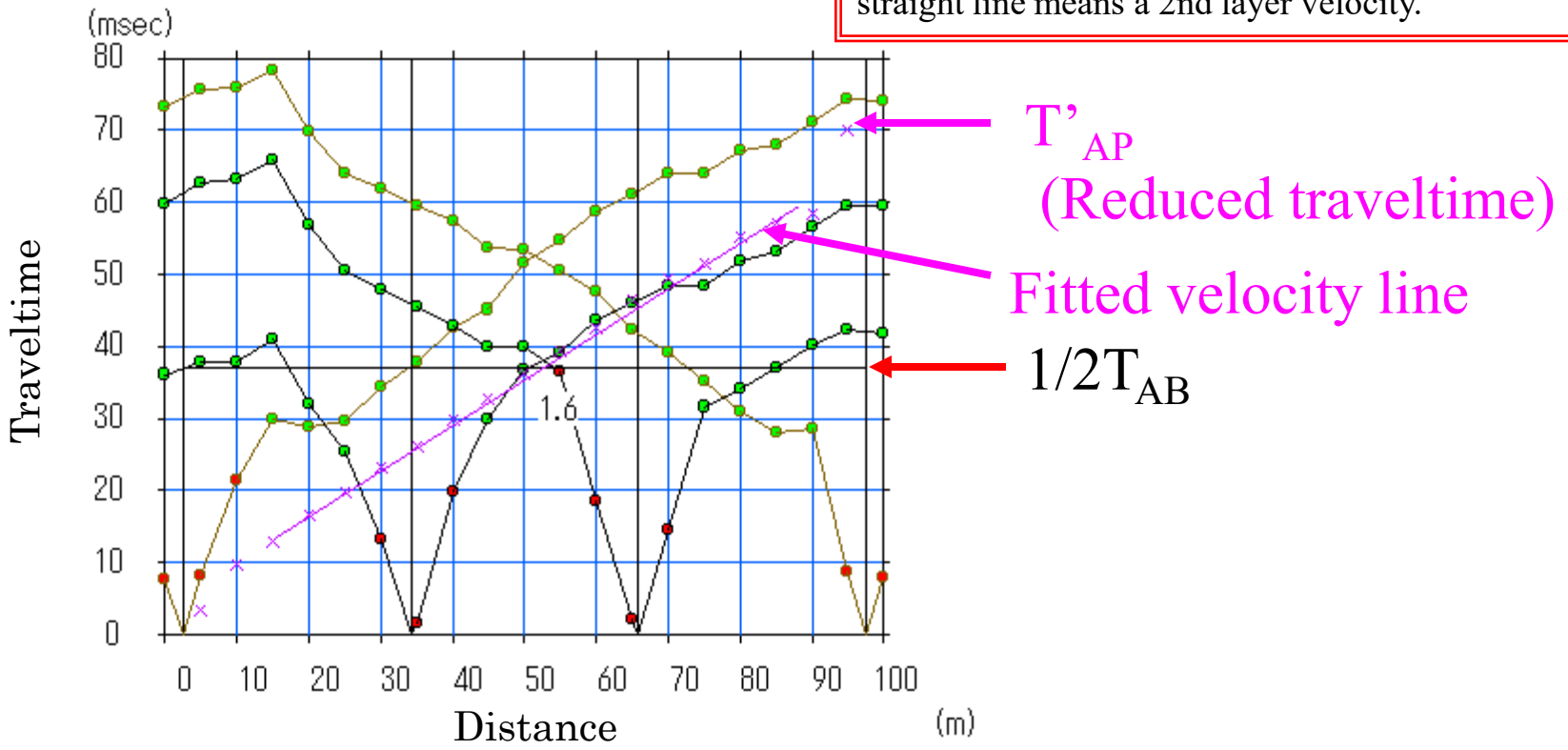


Intercept time from the both sides of traveltime curves should be same

Reciprocal method analysis procedure

Calculating reduced traveltime (T')

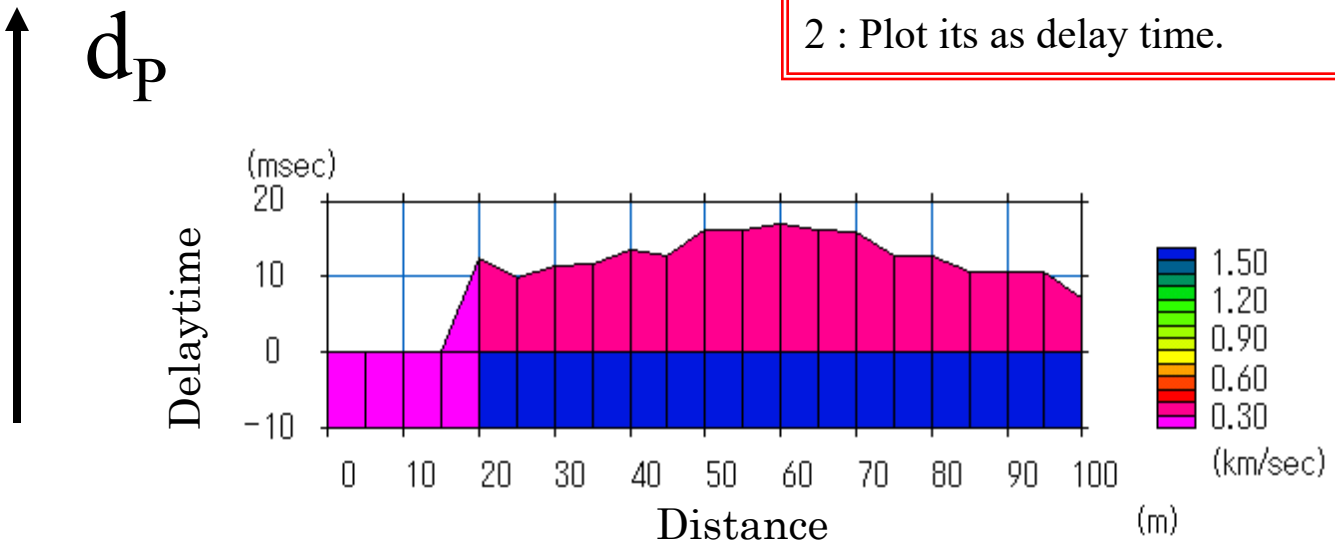
1 : Measure $1/2T_{AB}$ by a dividers and draw a line.
2 : Measure T_{AP}-T_{BP} by a dividers and divide it into halves,
3 : dot T'_{AP} below (or above) $1/2T_{AB}$.
4 : Fit a straight line on T'_{AP} dots. The slope of the straight line means a 2nd layer velocity.



Reciprocal method analysis procedure

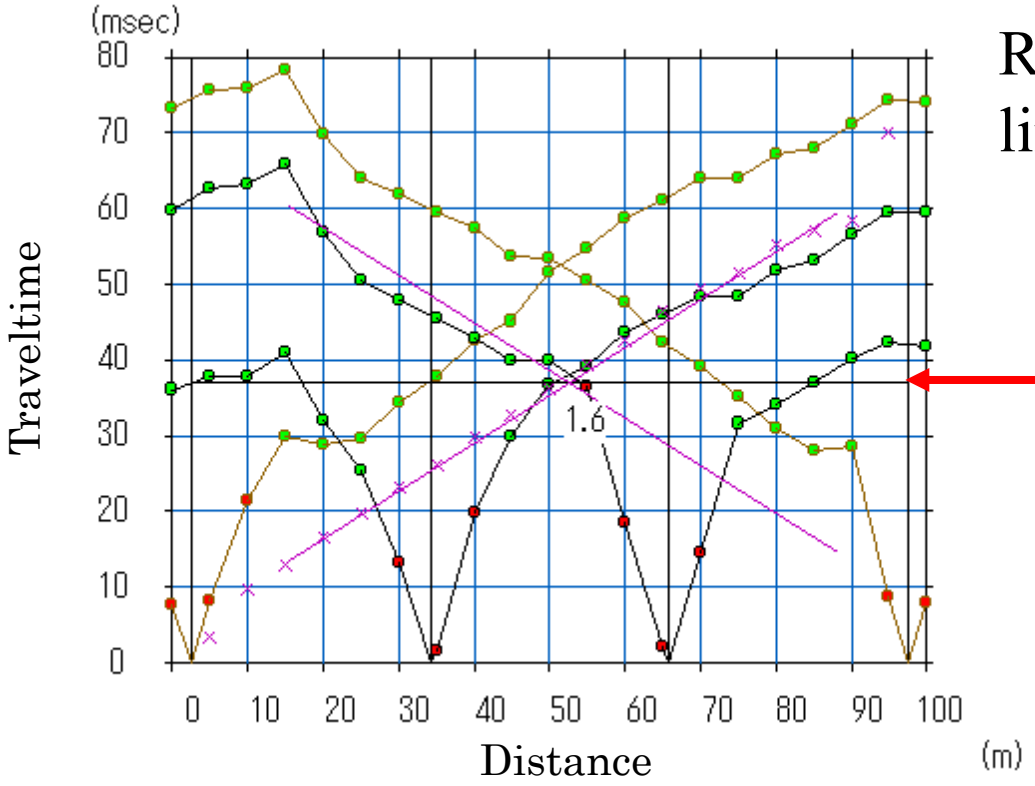
Calculating delay time

1 :Measure a time from the straight line fitted on T' dots to each traveltimes.
2 : Plot its as delay time.



Reciprocal method analysis procedure

Calculating T' (Reverse velocity line)

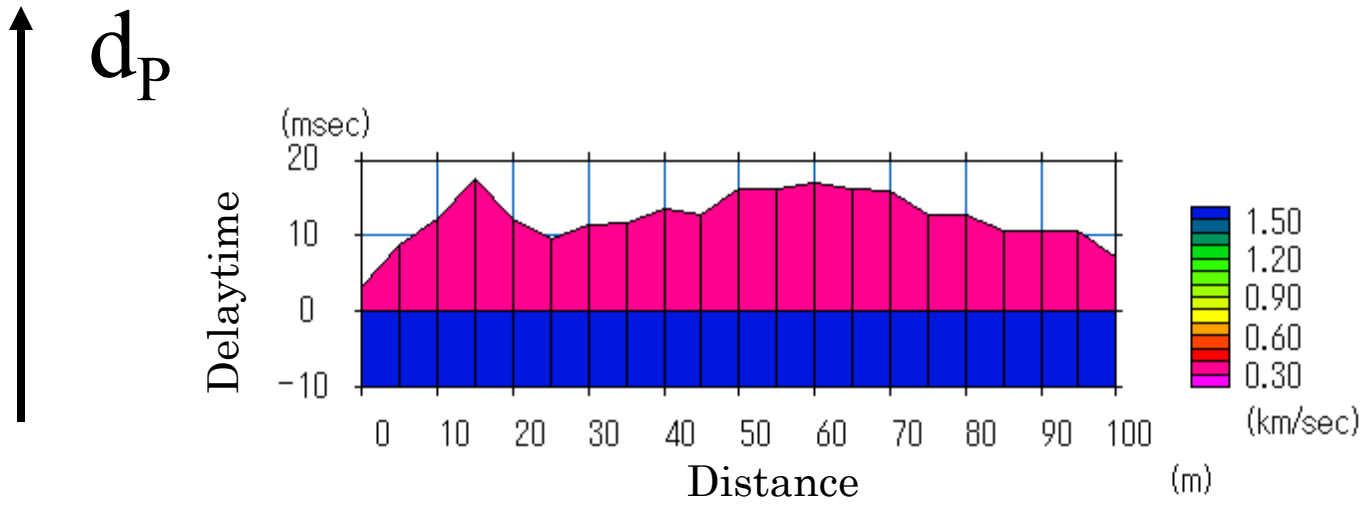


Reverse a velocity line based on $1/2T_{AB}$.

$1/2T_{AB}$

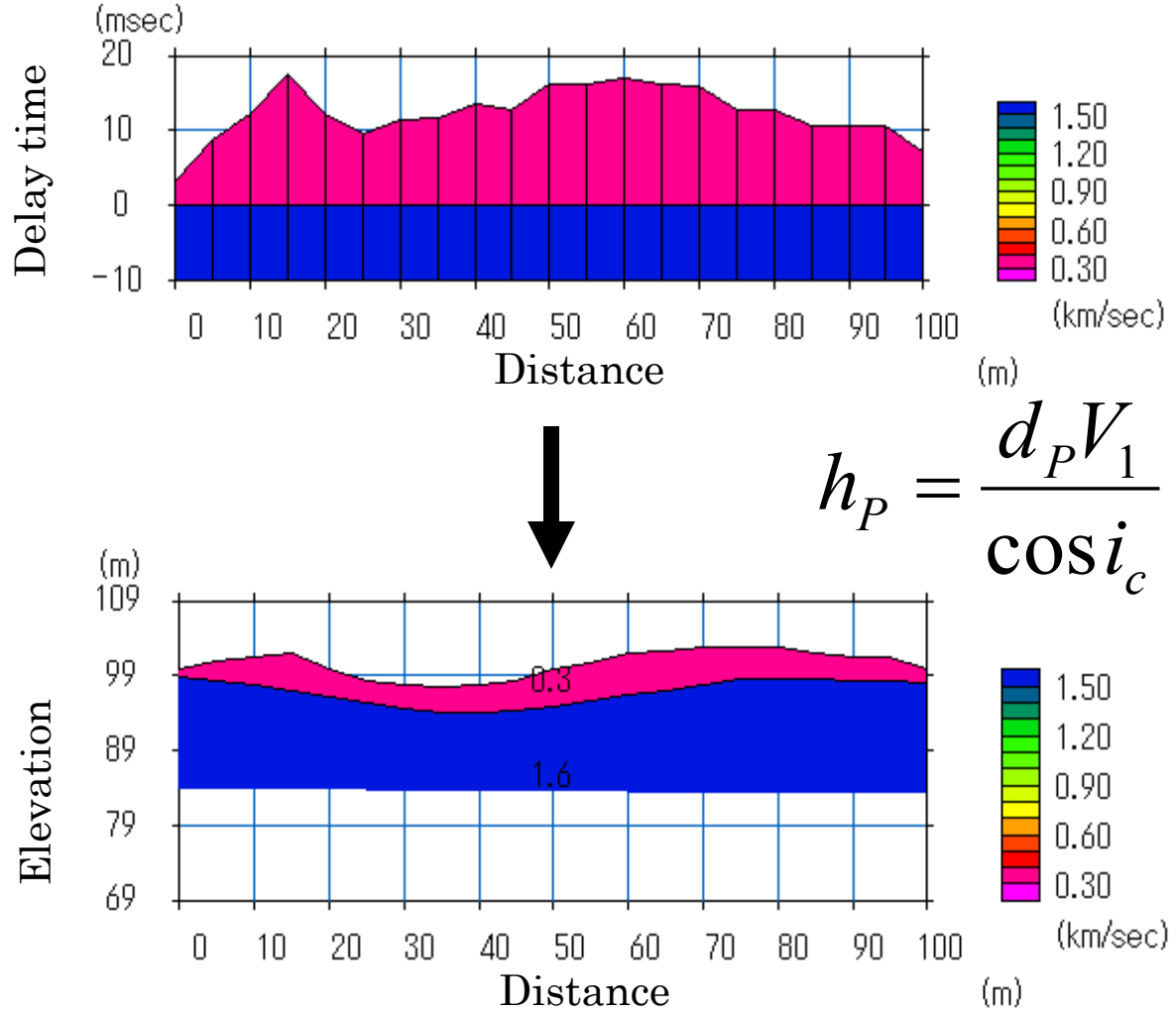
Reciprocal method analysis procedure

Calculating delay time



Reciprocal method analysis procedure

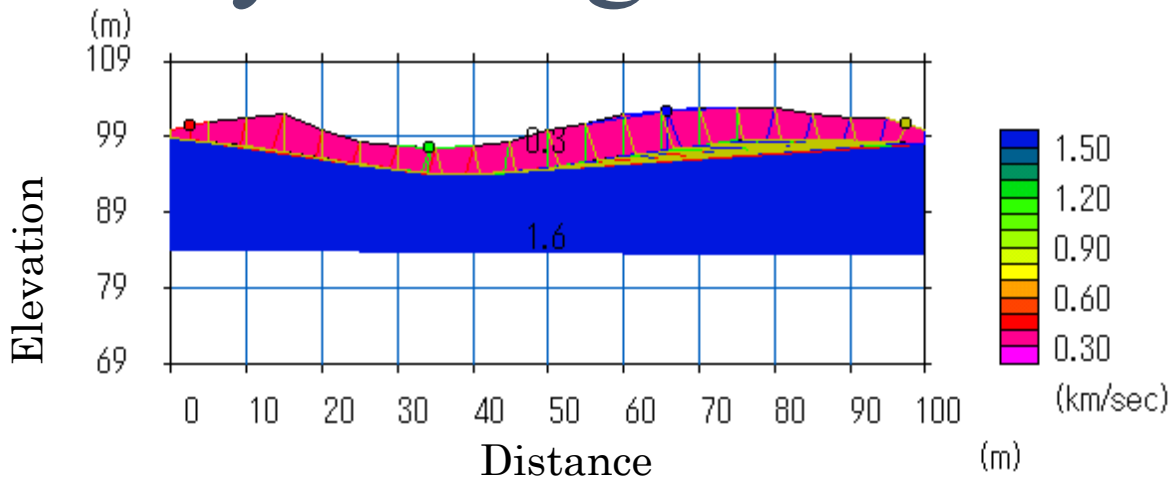
Calculating velocity model



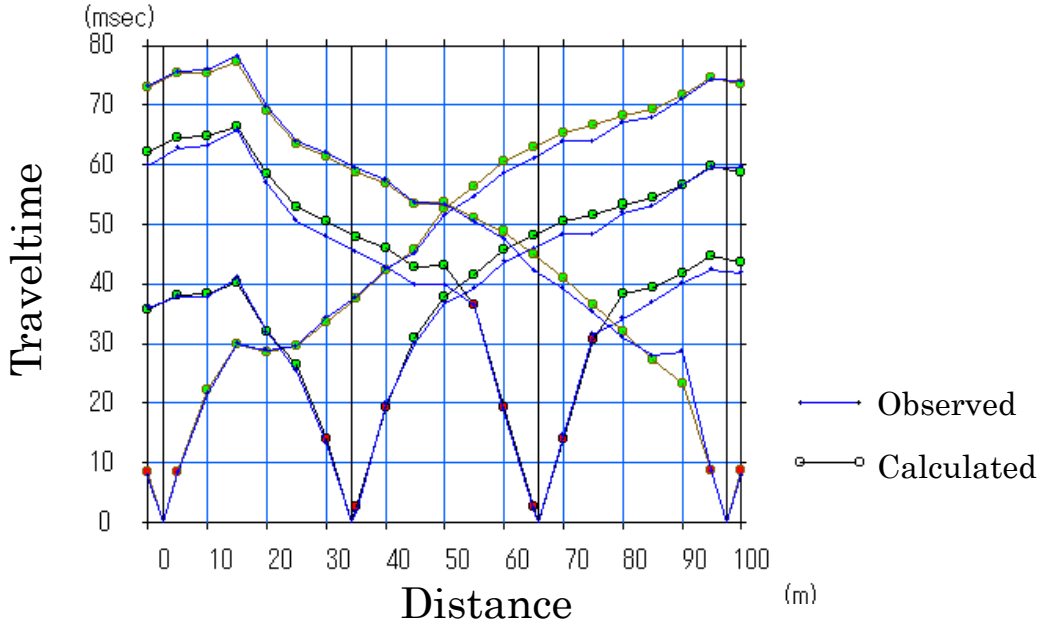
$$h_P = \frac{d_P V_1}{\cos i_c}$$

Raytracing

Raytracing

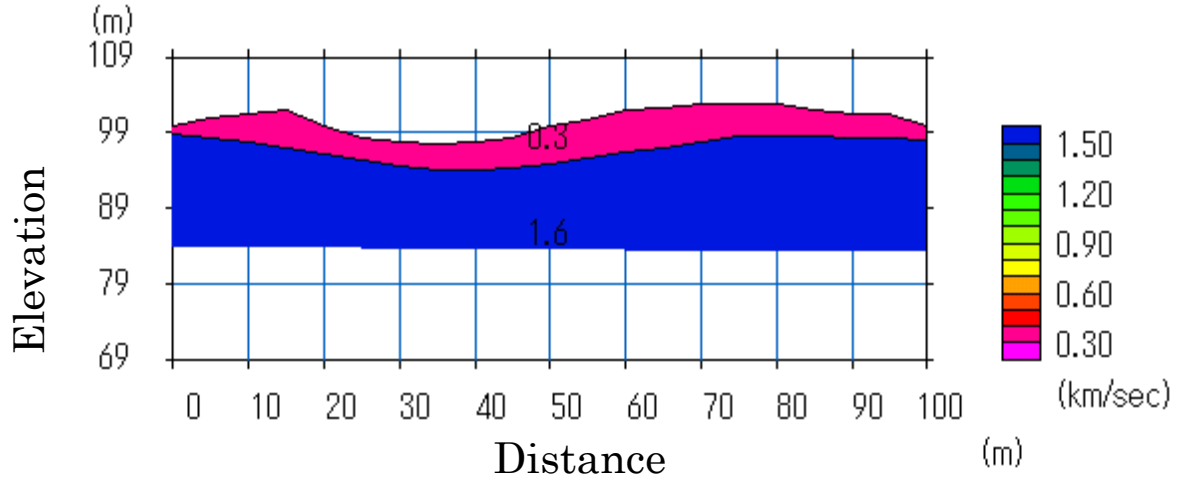


Comparison of calculated and observed traveltimes

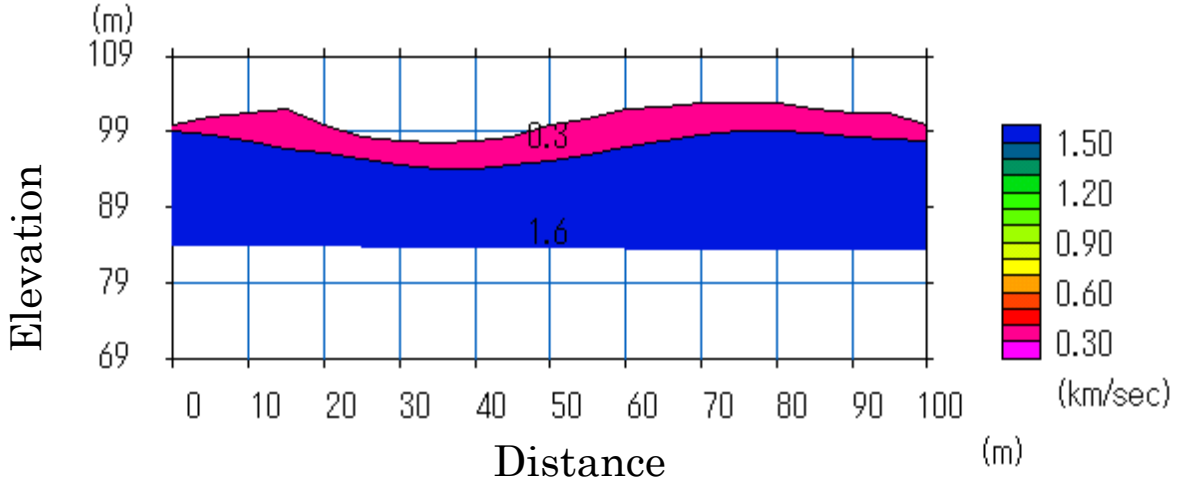


Correcting velocity model

Velocity model obtained from reciprocal method

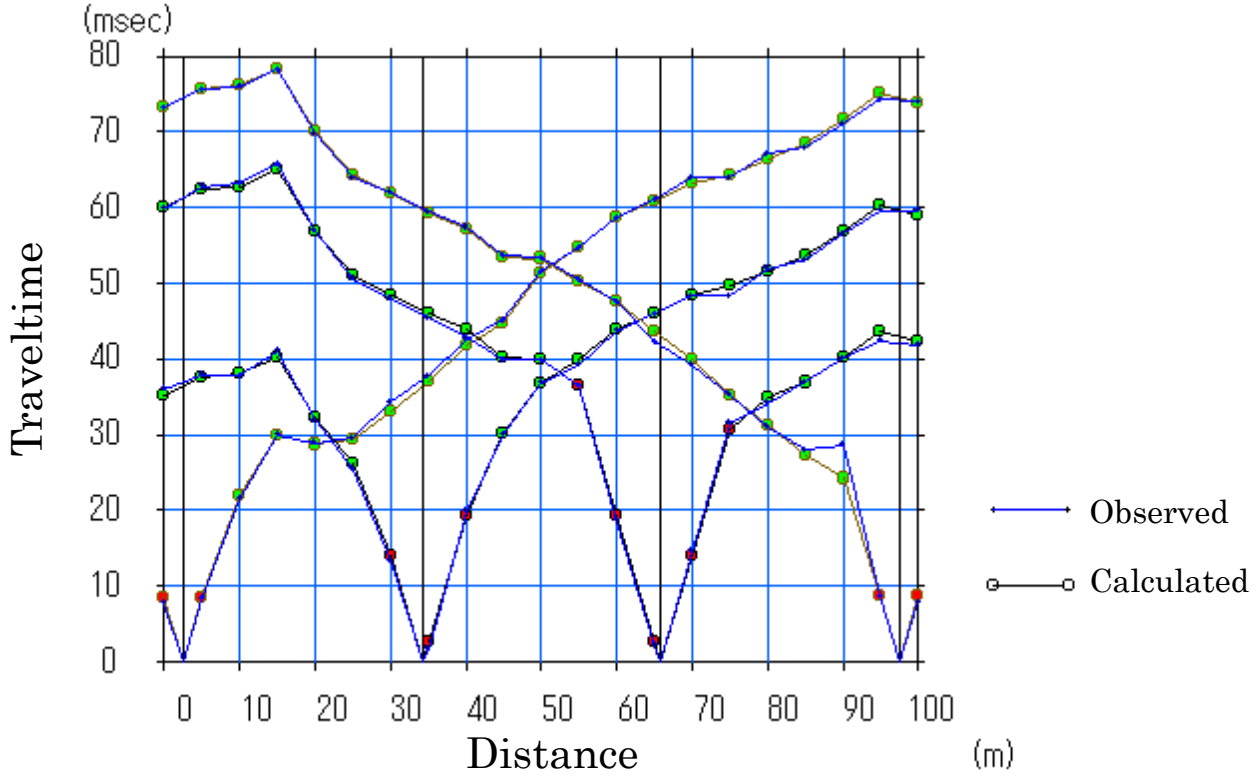


After correction

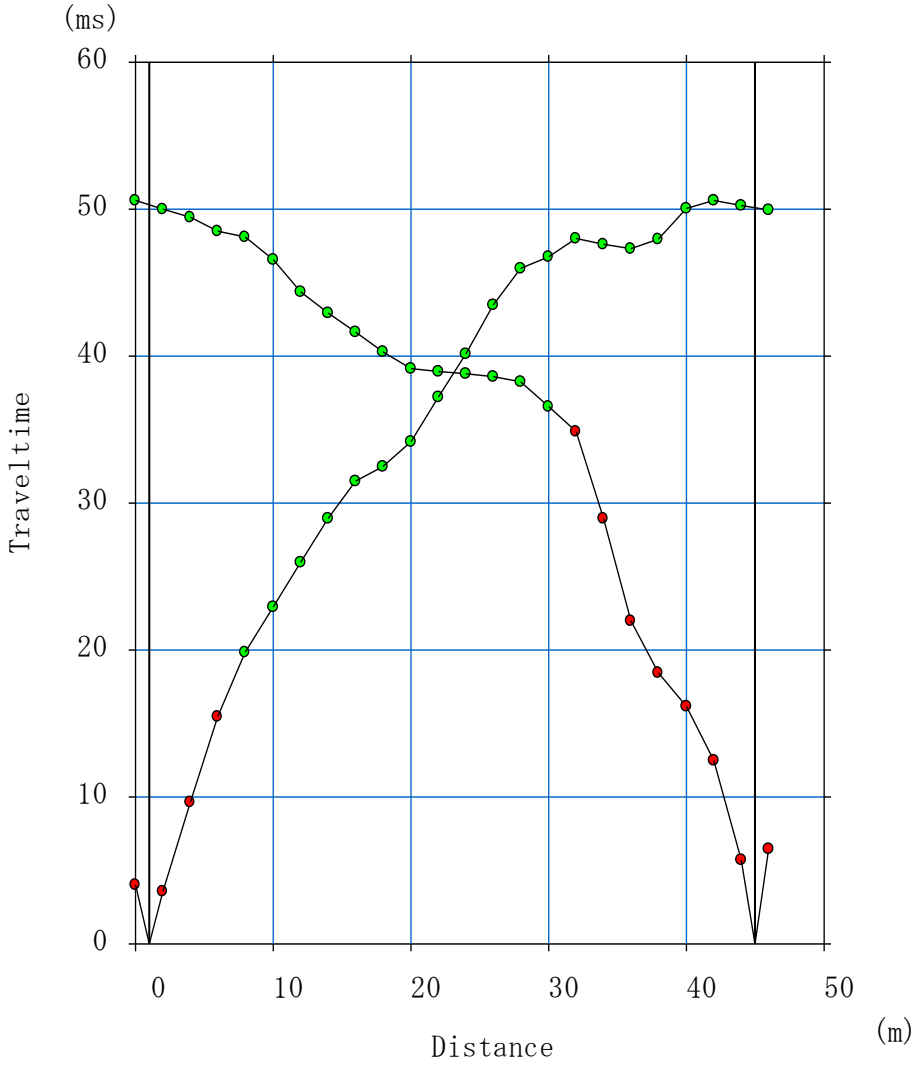


Correcting velocity model

Comparison of calculated and observed traveltimes (After correction)



Simple example of reciprocal method

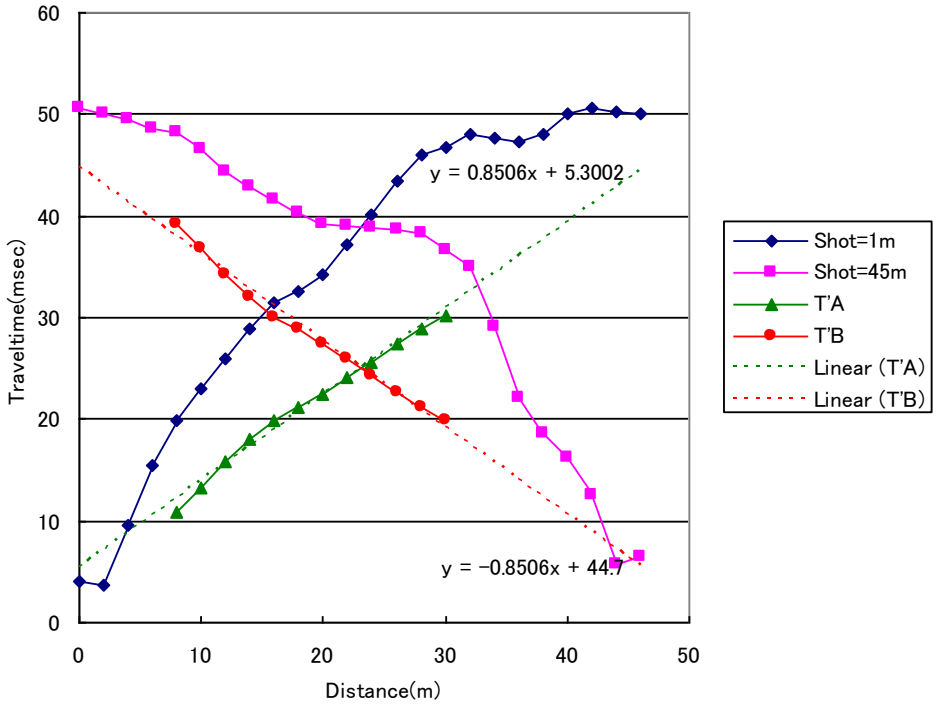


Simple example of reciprocal method Analysis using spread sheet

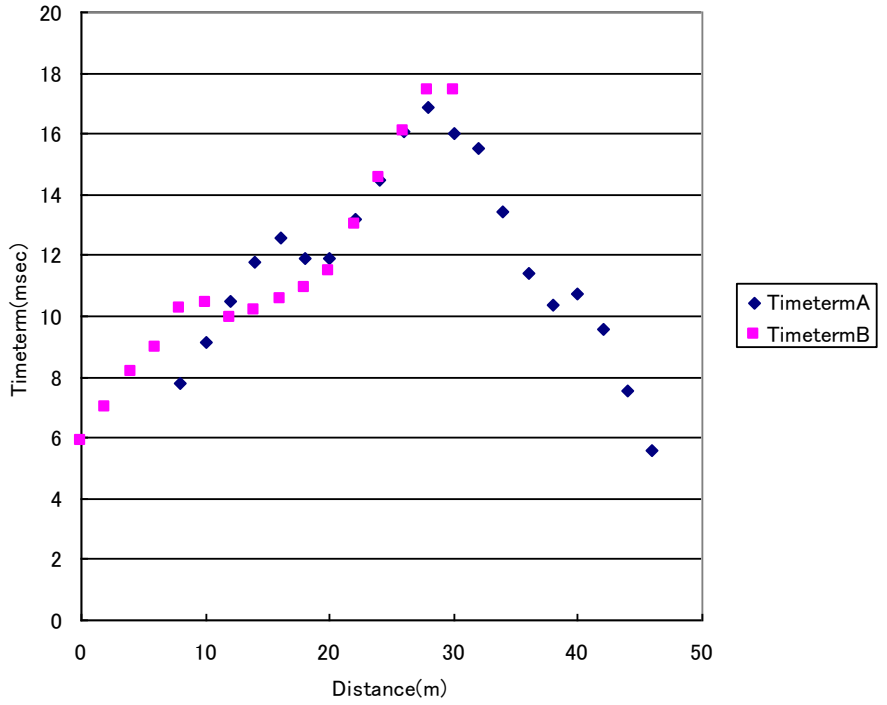
						V1=	0.3	V2=	1.1756		sin(ic)=	0.25518	cos(ic)=	0.966893566	
	Shot=1m	Shot=45m	TAB	1/2TAB	1/2TAB+(A−B)/2	T'A	T"A	1/2TAB+(B−A)/2	T'B	T"B	TimetermA	TimetermB	Timeterm	cos(ic)*v1	h
0	4.049999	50.609409	50	25	1.720295			48.279705		44.7		5.909409	5.909409	0.29006807	1.7141
2	3.599999	50.021172	50	25	1.7894135			48.2105865		42.999		7.022372	7.022372	0.29006807	2.037
4	9.652247	49.464176	50	25	5.0940355			44.9059645		41.298		8.166576	8.166576	0.29006807	2.3689
6	15.517414	48.552567	50	25	8.4824235			41.5175765		39.596		8.956167	8.956167	0.29006807	2.5979
8	19.885326	48.151424	50	25	10.866951	10.867	12.105	39.133049	39.133	37.895	7.780326	10.256224	9.018275	0.29006807	2.6159
10	22.939219	46.600403	50	25	13.169408	13.169	13.806	36.830592	36.831	36.194	9.133019	10.406403	9.769711	0.29006807	2.8339
12	25.993113	44.413956	50	25	15.7895785	15.79	15.507	34.2104215	34.21	34.493	10.485713	9.921156	10.203435	0.29006807	2.9597
14	28.963123	42.972939	50	25	17.995092	17.995	17.209	32.004908	32.005	32.792	11.754523	10.181339	10.967931	0.29006807	3.1814
16	31.494371	41.646736	50	25	19.9238175	19.924	18.91	30.0761825	30.076	31.09	12.584571	10.556336	11.570454	0.29006807	3.3562
18	32.532303	40.32053	50	25	21.1058865	21.106	20.611	28.8941135	28.894	29.389	11.921303	10.93133	11.426317	0.29006807	3.3144
20	34.224609	39.168789	50	25	22.52791	22.528	22.312	27.47209	27.472	27.688	11.912409	11.480789	11.696599	0.29006807	3.3928
22	37.218212	38.996376	50	25	24.110918	24.111	24.013	25.889082	25.889	25.987	13.204812	13.009576	13.107194	0.29006807	3.802
24	40.164642	38.823963	50	25	25.6703395	25.67	25.715	24.3296605	24.33	24.286	14.450042	14.538363	14.494203	0.29006807	4.2043
26	43.500004	38.65155	50	25	27.424227	27.424	27.416	22.575773	22.576	22.584	16.084204	16.06715	16.075677	0.29006807	4.663
28	46.000004	38.299145	50	25	28.8504295	28.85	29.117	21.1495705	21.15	20.883	16.883004	17.415945	17.149475	0.29006807	4.9745
30	46.80582	36.597015	50	25	30.1044025	30.104	30.818	19.8955975	19.896	19.182	15.98762	17.415015	16.701318	0.29006807	4.8445
32	48.014225	34.89489	50	25	31.5596675		32.519	18.4403325			15.494825		15.494825	0.29006807	4.4946
34	47.629608	29.000004	50	25	34.314802		34.221	15.685198			13.409008		13.409008	0.29006807	3.8895
36	47.347656	22.000002	50	25	37.673827		35.922	12.326173			11.425856		11.425856	0.29006807	3.3143
38	48.003109	18.500002	50	25	39.7515535		37.623	10.2484465			10.380109		10.380109	0.29006807	3.0109
40	50.078133	16.207413	50	25	41.93536		39.324	8.06464			10.753933		10.753933	0.29006807	3.1194
42	50.60651	12.522017	50	25	44.0422465		41.025	5.9577535			9.58111		9.58111	0.29006807	2.7792
44	50.296833	5.759991	50	25	47.268421		42.727	2.731579			7.570233		7.570233	0.29006807	2.1959
46	49.987156	6.500001	50	25	46.7435775		44.428	3.2564225			5.559356		5.559356	0.29006807	1.6126

Simple example of reciprocal method Analysis using spread sheet

Traveltime curve and reduced traveltime (T')

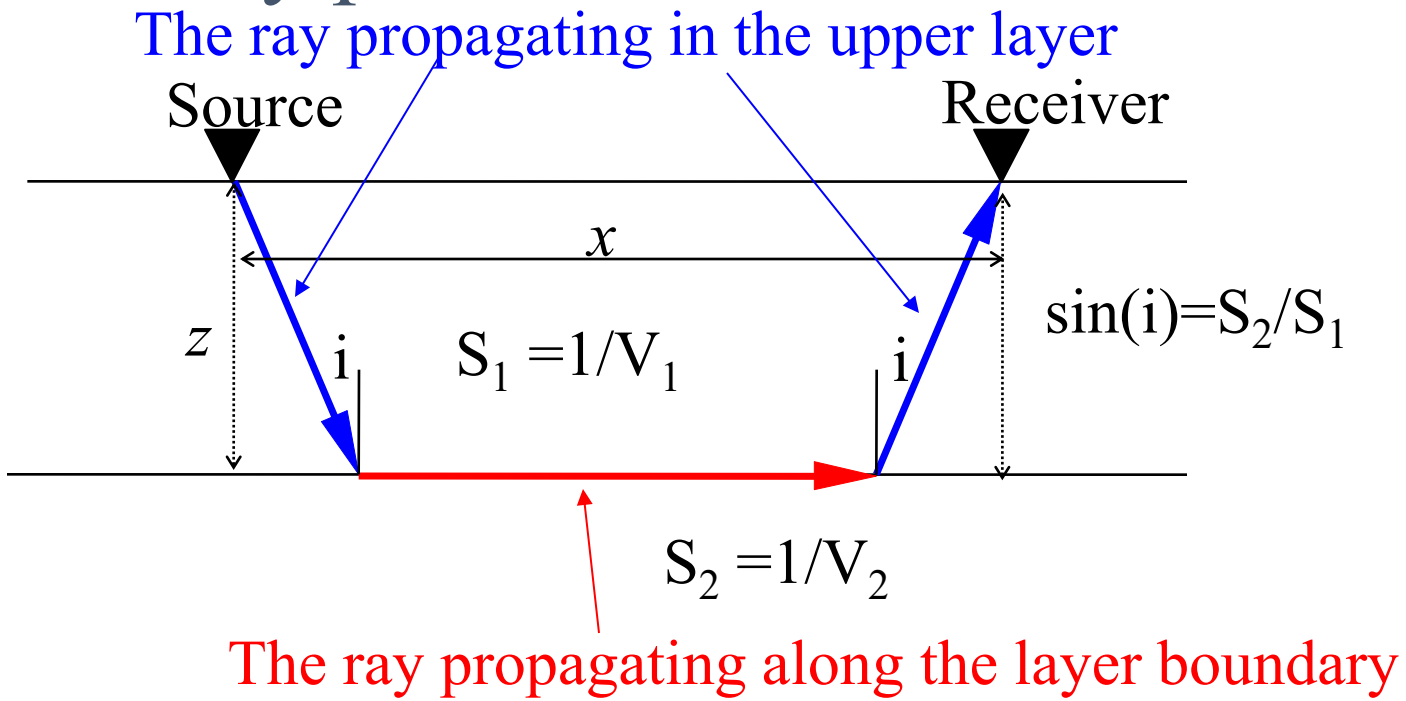


Delay time (time-term)



Time-term inversion (Automatic 2 layer analysis)

Refraction ray path

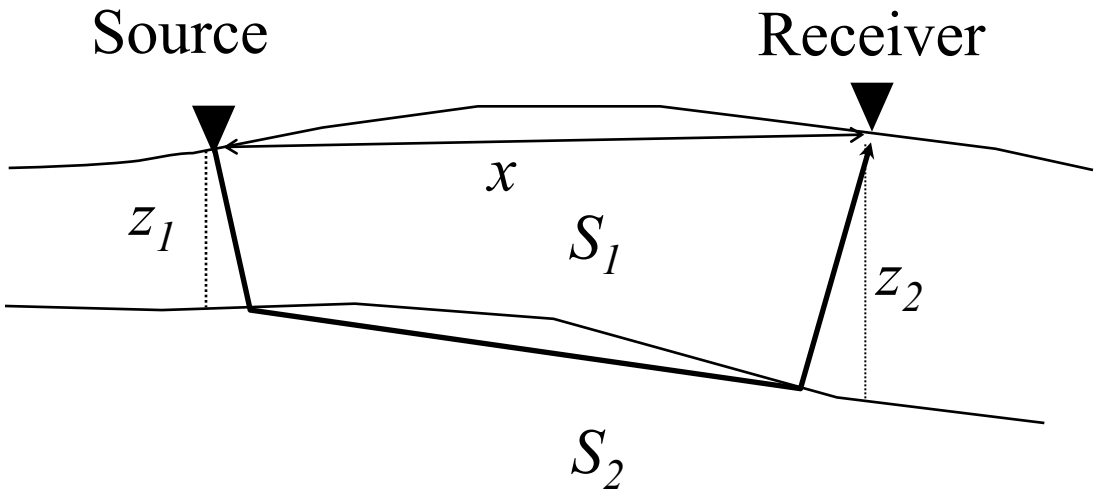


$$t = 2s_1 \cos(i) z + x s_2 \rightarrow t = 2cz + x s_2$$

$$c = s_1 \cos(i) \qquad z \text{ and } s_2 : \text{unknown}$$

Time-term inversion (Automatic 2 layer analysis)

Irregular layer boundary



$$t_i = c_1 z_1 + c_2 z_2 + x_i s_2$$

$$c_1 = c_2 = s_1 \cos(i)$$ *z_1,z_2 and s_2 : unknown*

Time-term inversion (Automatic 2 layer analysis)

Irregular layer boundary

$$t_i = \sum_{j=1}^{n} c_{ij} z_j + x_i s \qquad c_{ij} = s_1 \cos(i)$$

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & \cdot & c_{1n} & x_1 \\ c_{21} & c_{22} & c_{23} & \cdot & c_{2n} & x_2 \\ c_{31} & c_{32} & c_{33} & \cdot & c_{3n} & x_3 \\ c_{41} & c_{42} & c_{43} & \cdot & c_{4n} & x_4 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ c_{m1} & c_{m2} & c_{m3} & \cdot & c_{mn} & x_m \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ \cdot \\ z_n \\ s_2 \end{pmatrix} = \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ \cdot \\ t_m \end{pmatrix}$$

Linear least square method

m : Number of traveltimes

n : Number of receivers (depth to be inverted)

Time-term inversion (Automatic 2 layer analysis)

However,

$$t_i = \sum_{j=1}^{n} c_{ij} z_j + x_i s$$

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & \cdot & c_{1n} & x_1 \\ c_{21} & c_{22} & c_{23} & \cdot & c_{2n} & x_2 \\ c_{31} & c_{32} & c_{33} & \cdot & c_{3n} & x_3 \\ c_{41} & c_{42} & c_{43} & \cdot & c_{4n} & x_4 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ c_{m1} & c_{m2} & c_{m3} & \cdot & c_{mn} & x_m \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ \cdot \\ z_n \\ s_2 \end{pmatrix} = \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ \cdot \\ t_m \end{pmatrix}$$

$$c_{ij} = s_1 \cos(i)$$

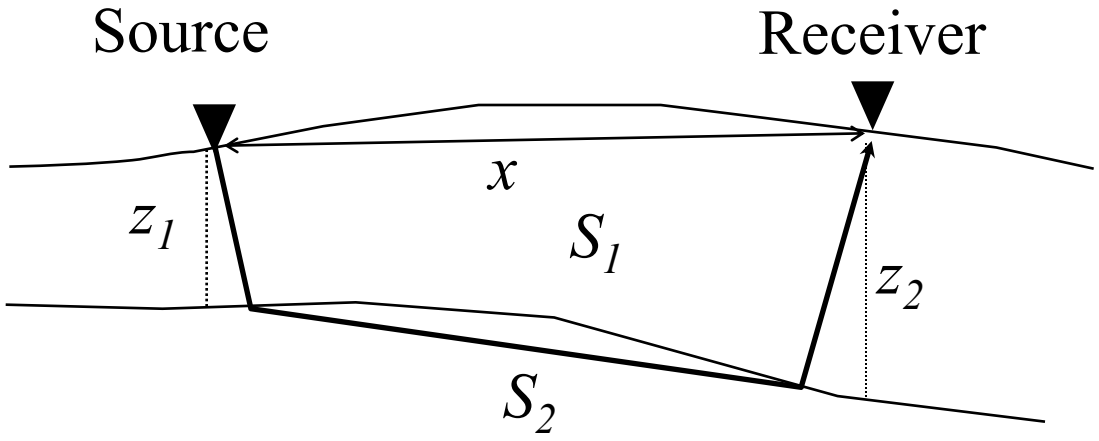
$$i = \arcsin \frac{S_2}{S_1}$$

S_2 cannot be calculated !

This problem is non-linear!

Time-term inversion (Automatic 2 layer analysis)

Introduce time-term!



$$t_i = c_1 z_1 + c_2 z_2 + x_i s_2$$

$$\tau_1 = c_1 z_1 = s_1 \cos(i) \cdot z_1$$

$$\tau_2 = c_2 z_2 = s_1 \cos(i) \cdot z_2$$

Time-term

$$t_i = \tau_1 + \tau_2 + x_i s_2$$

Time-term inversion (Automatic 2 layer analysis)

Introduce time-term!

$$t_i = \sum_{j=1}^{n} \tau_j + x_i s$$

$c_{ij} = 0$ or 1 !

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & \cdot & c_{1n} & x_1 \\ c_{21} & c_{22} & c_{23} & \cdot & c_{2n} & x_2 \\ c_{31} & c_{32} & c_{33} & \cdot & c_{3n} & x_3 \\ c_{41} & c_{42} & c_{43} & \cdot & c_{4n} & x_4 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ c_{m1} & c_{m2} & c_{m3} & \cdot & c_{mn} & x_m \end{pmatrix} \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \cdot \\ \tau_n \\ s_2 \end{pmatrix} = \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ \cdot \\ t_m \end{pmatrix}$$

Linear least square method

m : Number of traveltimes

n : Number of receivers (depth to be inverted)

Time-term inversion (Automatic 2 layer analysis)

Simple case

2 sources : τ_{s1} and τ_{s2}

24 receivers : τ_{r1} and τ_{r24}

48 traveltimes

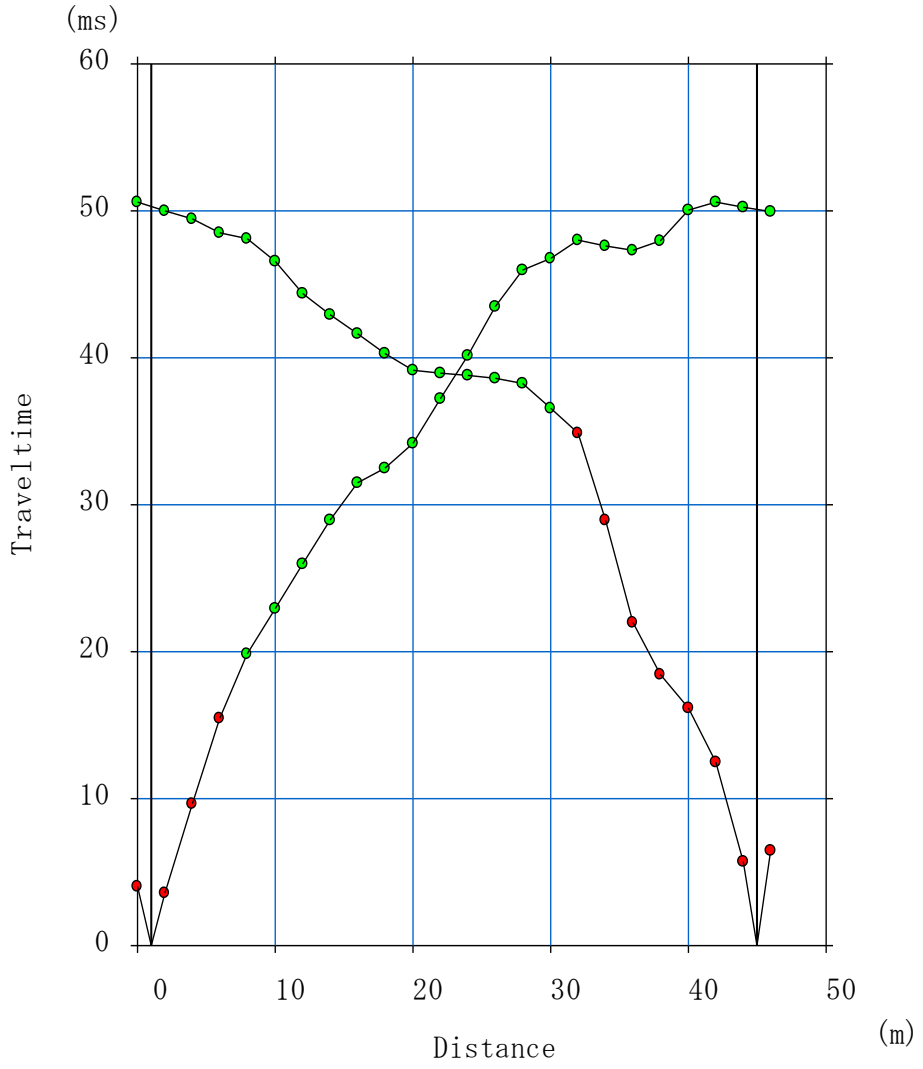
→ 27 unknowns
(2sources, 24 receivers, S_2)
48 data

$$\begin{pmatrix} 1 & 0 & 1 & \cdot & 0 & x_1 \\ 1 & 0 & 0 & \cdot & 0 & x_2 \\ 1 & 0 & 0 & \cdot & 0 & x_3 \\ 1 & 0 & 0 & \cdot & 0 & x_4 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 1 & 0 & \cdot & 1 & x_m \end{pmatrix} \begin{pmatrix} \tau_{s1} \\ \tau_{s2} \\ \tau_{r1} \\ \cdot \\ \tau_{r24} \\ s_2 \end{pmatrix} = \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ \cdot \\ t_{48} \end{pmatrix}$$

Linear least square method !

$$z_j = \frac{\tau_j}{s_1 \cos(i)}$$

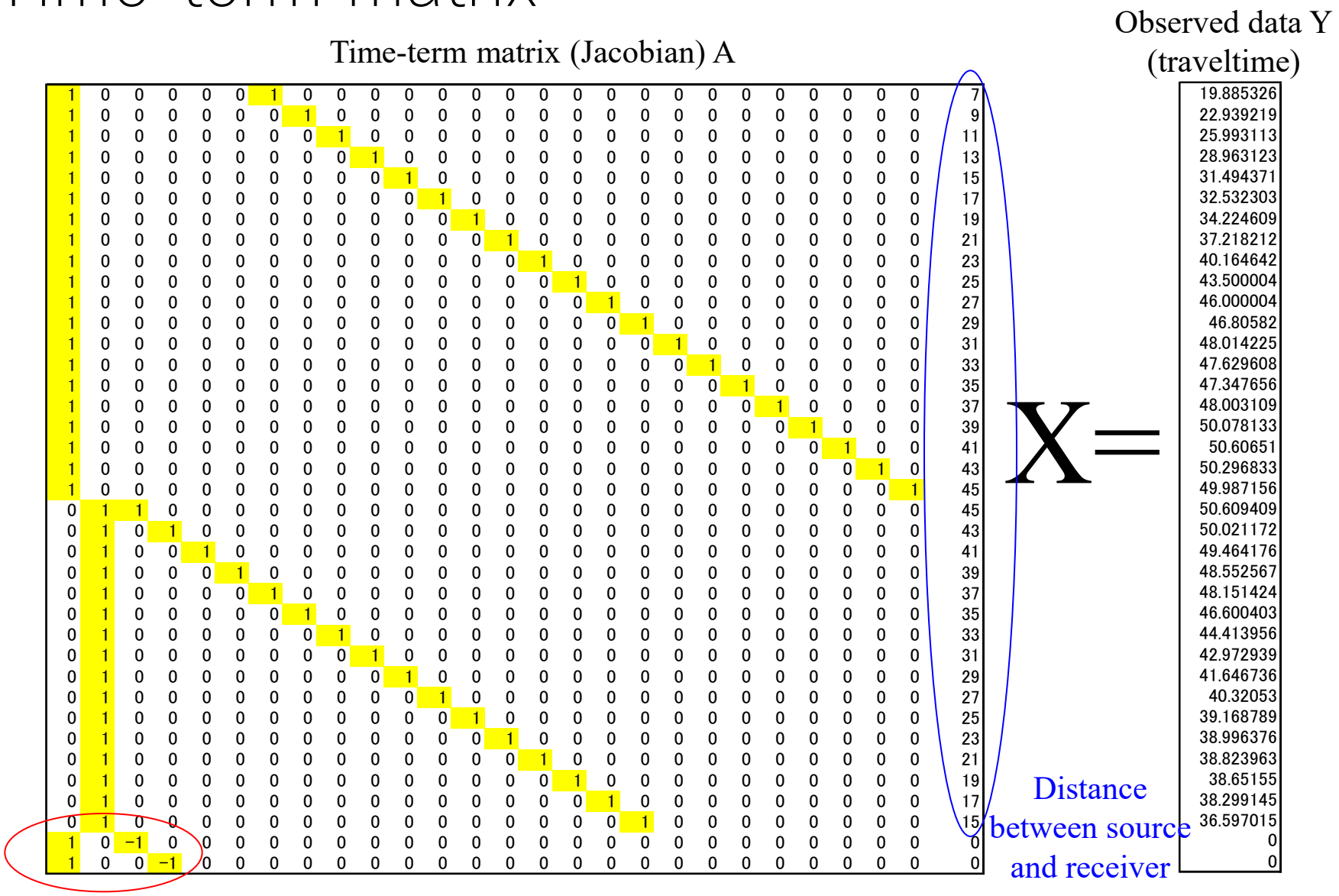
Simple example of time-term inversion



$$\begin{pmatrix} 1 & 0 & 1 & \cdot & 0 & x_1 \\ 1 & 0 & 0 & \cdot & 0 & x_2 \\ 1 & 0 & 0 & \cdot & 0 & x_3 \\ 1 & 0 & 0 & \cdot & 0 & x_4 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 1 & 0 & \cdot & 1 & x_m \end{pmatrix} \begin{pmatrix} \tau_{s1} \\ \tau_{s2} \\ \tau_{r1} \\ \cdot \\ \tau_{r24} \\ s_2 \end{pmatrix} = \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ \cdot \\ t_{48} \end{pmatrix}$$

$$AX = Y$$

Simple example of time-term inversion
Time-term matrix



Simple example of time-term inversion
Linear least square method

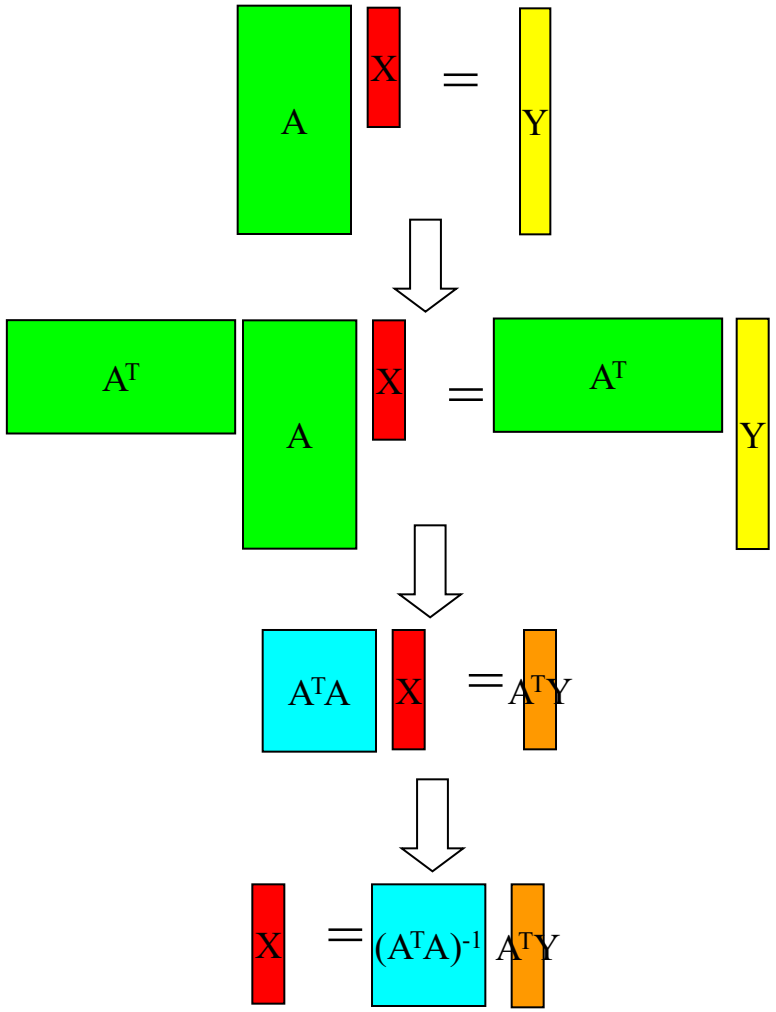
$$AX = Y$$

$$\Downarrow$$

$$A^T AX = A^T Y$$

$$\Downarrow$$

$$X = \left(A^T A\right)^{-1} A^T Y$$



Simple example of time-term inversion
Linear least square method

$$A^T A X = A^T Y$$

$A^T A$

22	0	−1	−1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	520
0	16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	480
−1	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	45
−1	1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	43
0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	41
0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	39
1	1	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	44
1	1	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	44
1	1	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	44
1	1	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	44
1	1	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	44
1	1	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	44
1	1	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	44
1	1	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	44
1	1	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	44
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	44
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	44
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	44
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	44
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	31
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	33
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	35
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	37
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	39
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	41
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	43
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	45
520	480	45	43	41	39	44	44	44	44	44	44	44	44	44	44	44	44	31	33	35	37	39	41	43	45	31940

X=

$A^T Y$

801.683976
693.29015
50.609409
50.021172
49.464176
48.552567
68.03675
69.539622
70.407069
71.936062
73.141107
72.852833
73.393398
76.214588
78.988605
82.151554
84.299149
83.402835
48.014225
47.629608
47.347656
48.003109
50.078133
50.60651
50.296833
49.987156
44492.2895

Simple example of time-term inversion
Linear least square method

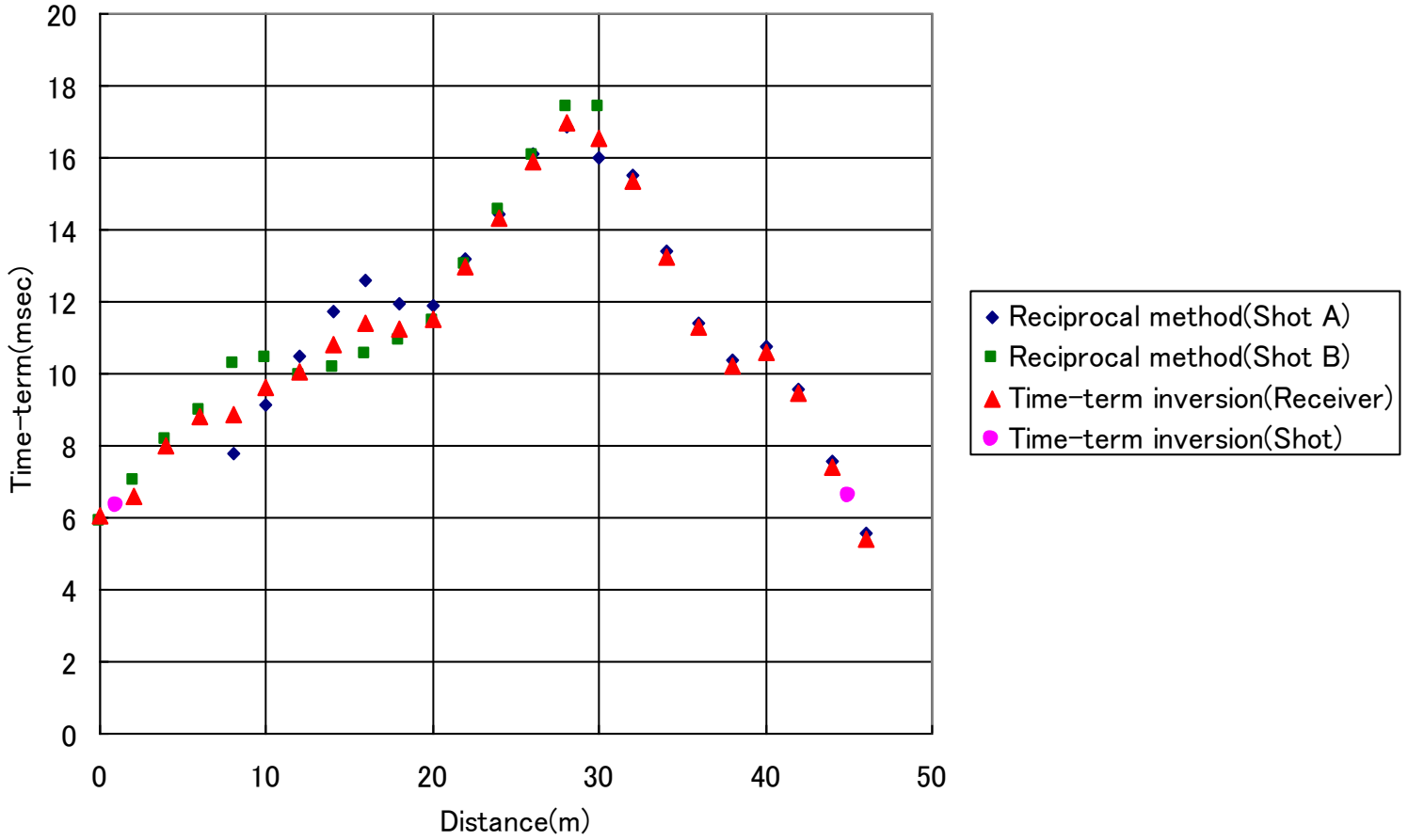
$$X = \left(A^T A\right)^{-1} A^T Y$$

X = $(A^TA)^{-1}$ A^TY =

X	$(A^TA)^{-1}$	A^TY
6.31717148	0.6 0.6 0.3 0.3 0 −0 −0 −0 −0 −0 −0 −0 −0 −0 −0 −0 −0 −0 −0 −0 0 0 0.1 0.1 0.1 −0.02	801.683976
6.5930987	0.6 0.9 0.4 0.4 0 0 −0 −0 −0 −0 −0 −0 −0 −0 −0 −0 −0 −0 0.1 0.1 0.2 0.2 0.3 0.3 0.4 0.4 −0.02	693.29015
6.03917368	0.3 0.4 0.8 0.3 0.3 0.3 −0 −0 −0 −0 −0 −0 −0 −0 −0 −0 −0 −0 0.2 0.2 0.2 0.3 0.3 0.3 0.4 0.4 −0.02	50.609409
6.59516928	0.3 0.4 0.3 0.8 0.3 0.2 −0 −0 −0 −0 −0 −0 −0 −0 −0 −0 −0 −0 0.2 0.2 0.2 0.2 0.3 0.3 0.3 0.4 −0.02	50.021172
8.01639928	0 0 0.3 0.3 1.5 0.5 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.4 0.4 0.4 0.5 0.5 0.5 0.5 0.6 −0.01	49.464176
8.80501847	−0 0 0.3 0.2 0.5 1.4 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.4 0.4 0.4 0.4 0.4 0.5 0.5 0.5 −0.01	48.552567
8.86072975	−0 −0 −0 −0 0.3 0.3 0.8 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.2 2E−17	68.03675
9.61216575	−0 −0 −0 −0 0.3 0.3 0.3 0.8 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.2 2E−17	69.539622
10.0458893	−0 −0 −0 −0 0.3 0.3 0.3 0.3 0.8 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.2 2E−17	70.407069
10.8103858	−0 −0 −0 −0 0.3 0.3 0.3 0.3 0.3 0.8 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.2 2E−17	71.936062
11.4129083	−0 −0 −0 −0 0.3 0.3 0.3 0.3 0.3 0.3 0.8 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.2 2E−17	73.141107
11.2687713	−0 −0 −0 −0 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.8 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.2 2E−17	72.852833
11.5390537	−0 −0 −0 −0 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.8 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.2 2E−17	73.393398
12.9496488	−0 −0 −0 −0 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.8 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.2 2E−17	76.214588
14.3366572	−0 −0 −0 −0 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.8 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.2 2E−17	78.988605
15.9181317	−0 −0 −0 −0 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.8 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.2 2E−17	82.151554
16.9919292	−0 −0 −0 −0 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.8 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.2 2E−17	84.299149
16.5437723	−0 −0 −0 −0 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.8 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.2 2E−17	83.402835
15.3435165	−0 0.1 0.2 0.2 0.4 0.4 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 1.4 0.5 0.5 0.5 0.5 0.6 0.6 0.6 −0.01	48.014225
13.2586713	−0 0.1 0.2 0.2 0.4 0.4 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.5 1.5 0.5 0.5 0.6 0.6 0.6 0.6 −0.01	47.629608
11.2764911	−0 0.2 0.2 0.2 0.4 0.4 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.5 0.5 1.5 0.6 0.6 0.6 0.7 0.7 −0.01	47.347656
10.2317159	0 0.2 0.3 0.2 0.5 0.4 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.5 0.5 0.6 1.6 0.6 0.7 0.7 0.7 −0.02	48.003109
10.6065117	0 0.3 0.3 0.3 0.5 0.4 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.5 0.6 0.6 0.6 1.7 0.7 0.8 0.8 −0.02	50.078133
9.43466049	0.1 0.3 0.3 0.3 0.5 0.5 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.6 0.6 0.6 0.7 0.7 1.8 0.8 0.8 −0.02	50.60651
7.42475529	0.1 0.4 0.4 0.3 0.5 0.5 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.6 0.6 0.7 0.7 0.8 0.8 1.8 0.9 −0.02	50.296833
5.4148501	0.1 0.4 0.4 0.4 0.6 0.5 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.6 0.6 0.7 0.7 0.8 0.8 0.9 1.9 −0.02	49.987156
0.8501141	−0 −0 −0 −0 −0 −0 0 0 0 0 0 0 0 0 0 0 0 0 −0 −0 −0 −0 −0 −0 −0 −0 9E−04	44492.2895

Slowness of 2nd layer

Simple example of time-term inversion
Comparison with reciprocal method



Spectrum Inversion for Site Amplification

Time-term method

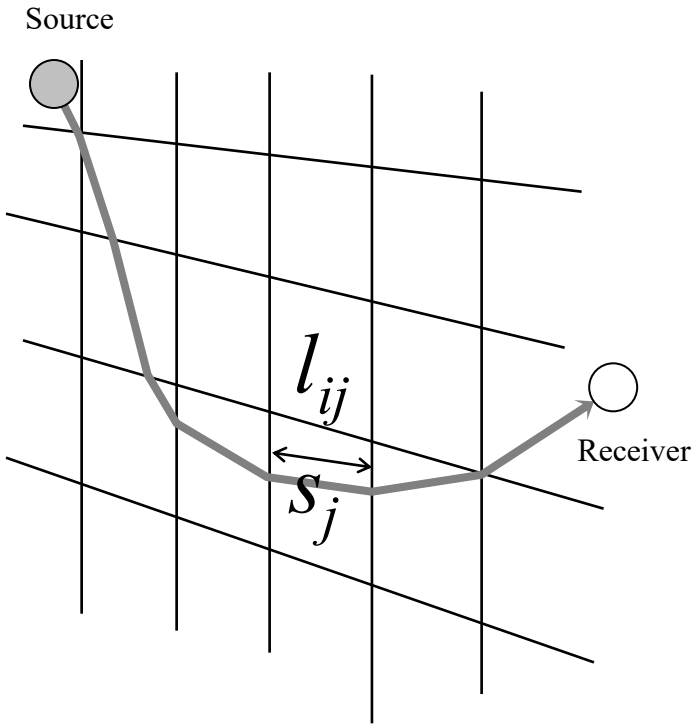
$$t_i = \tau_1 + \tau_2 + x_i s_2$$

Convolution model

$$O_{ij}(f) = S_i(f) * \frac{1}{x_{ij}} P_{ij}(f) * G_j(f)$$

$$\log O_{ij}(f) = \log S_i(f) - x_{ij} \log P_{ij}(f) + \log G_j(f)$$

Traveltime tomography
(Raypath and traveltime discretization)



$$t_i = \int_X \frac{dX}{\upsilon(X)} = \int_X s(X)dX$$

discretization

$$t_i = s_1 l_{i1} + s_2 l_{i2} + s_3 l_{i3} + s_4 l_{i4} + \cdots + s_N l_{iN}$$

$$t_i = \sum_{j=1}^{N} s_j l_{ij}$$

$$s = \frac{1}{\upsilon}$$

υ:velocity
s:slowness

Traveltime tomography

M simultaneous equations(M traveltime, N unknown)

$$
\begin{aligned}
t_1 &= l_{11}s_1 + l_{12}s_2 + \cdots + l_{1N}s_N \\
t_2 &= l_{21}s_1 + l_{22}s_2 + \cdots + l_{2N}s_N \\
t_3 &= l_{31}s_1 + l_{32}s_2 + \cdots + l_{3N}s_N \\
&\vdots \\
t_M &= l_{M1}s_1 + l_{M2}s_2 + \cdots + l_{MN}s_N
\end{aligned}
$$

Matrix notation

$$
LS = \begin{pmatrix} l_{11} & l_{12} & \cdot & l_{1N} \\ l_{21} & l_{22} & \cdot & l_{2N} \\ l_{31} & l_{32} & \cdot & l_{3N} \\ \cdot & \cdot & \cdot & \cdot \\ l_{M1} & l_{M2} & \cdot & l_{MN} \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \\ \cdot \\ s_N \end{pmatrix} = \begin{pmatrix} t_1 \\ t_2 \\ \cdot \\ \cdot \\ t_M \end{pmatrix} = T
$$

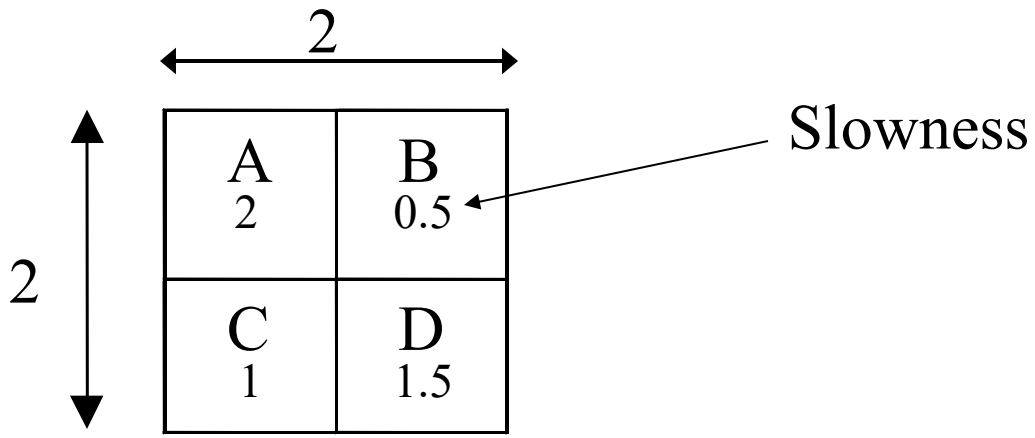
Raypaths model Traveltime

→ Least Square Method

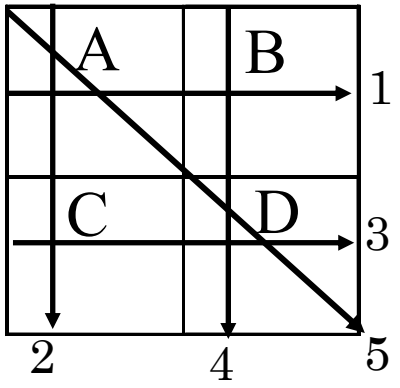
Generally M>N

Traveltime tomography example

4 cells (unknown)



5 ray-path (data)



Observed traveltime

$$T=\begin{pmatrix}t_1\\t_2\\t_3\\t_4\\t_5\end{pmatrix}=\begin{pmatrix}2+0.5\\2+1\\1+1.5\\0.5+1.5\\2\sqrt{2}+1.5\sqrt{2}\end{pmatrix}=\begin{pmatrix}2.5\\3\\2.5\\2\\4.949747\end{pmatrix}$$

Jacobian matrix A (Ray length passing through each cell)

$$L=\begin{pmatrix}1&1&0&0\\1&0&1&0\\0&0&1&1\\0&1&0&1\\\sqrt{2}&0&0&\sqrt{2}\end{pmatrix}$$

$$t_i = s_1 l_{i1} + s_2 l_{i2} + s_3 l_{i3} + s_4 l_{i4} + \cdots + s_N l_{iN}$$

$$\frac{\partial t_i}{\partial s_j} = l_{ij}$$

Equation to be solved

$$LS=\begin{pmatrix}1&1&0&0\\1&0&1&0\\0&0&1&1\\0&1&0&1\\\sqrt{2}&0&0&\sqrt{2}\end{pmatrix}\begin{pmatrix}s_1\\s_2\\s_3\\s_4\end{pmatrix}=\begin{pmatrix}2.5\\3\\2.5\\2\\4.949747\end{pmatrix}=T$$

Normal equation

$$L^T LS = \begin{pmatrix} 4 & 1 & 1 & 2 \\ 1 & 2 & 0 & 1 \\ 1 & 0 & 2 & 1 \\ 2 & 1 & 1 & 4 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} = \begin{pmatrix} 12.5 \\ 4.5 \\ 5.5 \\ 11.5 \end{pmatrix} = L^T T$$

Solve it !

$$S^T = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \end{pmatrix} = \begin{pmatrix} 2 & 0.5 & 1 & 1.5 \end{pmatrix}$$

Traveltime tomography

Simultaneous equations $LS = T$

L is the function of S $L(S)S = T \longrightarrow$ Non-linear problem

Initial model S_0 —Raytracing→ L_0 Jacobian matrix L is ray-paths!

S_0 → T_0^c Theoretical traveltimes

then $\Delta T_0 = T^O - T_0^C = T^O - L_0 S_0$

Calculate correction $L_0 \Delta S_0 = \Delta T_0$

Correct model $S_1 = S_0 + \Delta S_0$

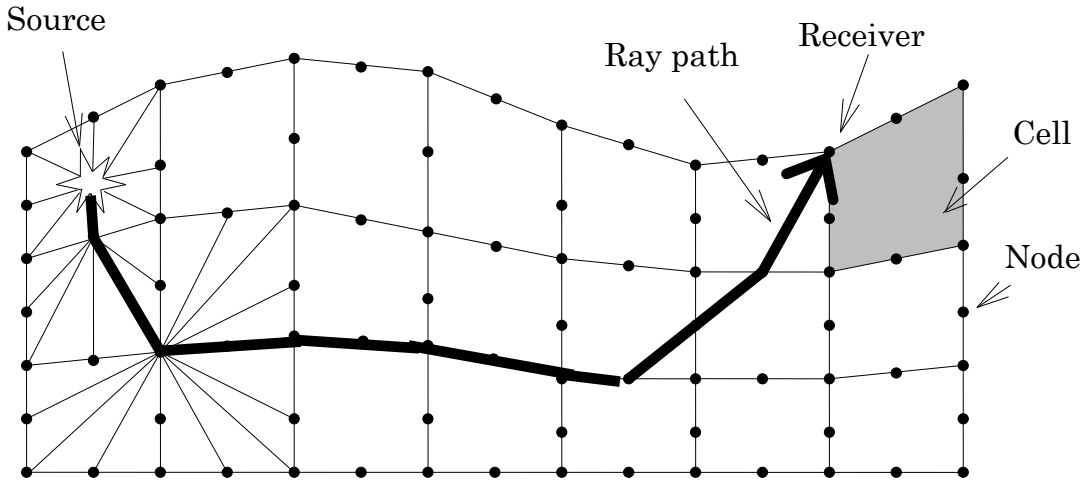
In the K_{th} iteration

$$\Delta T_k = T^O - T_k^C = T^O - L_k S_k$$

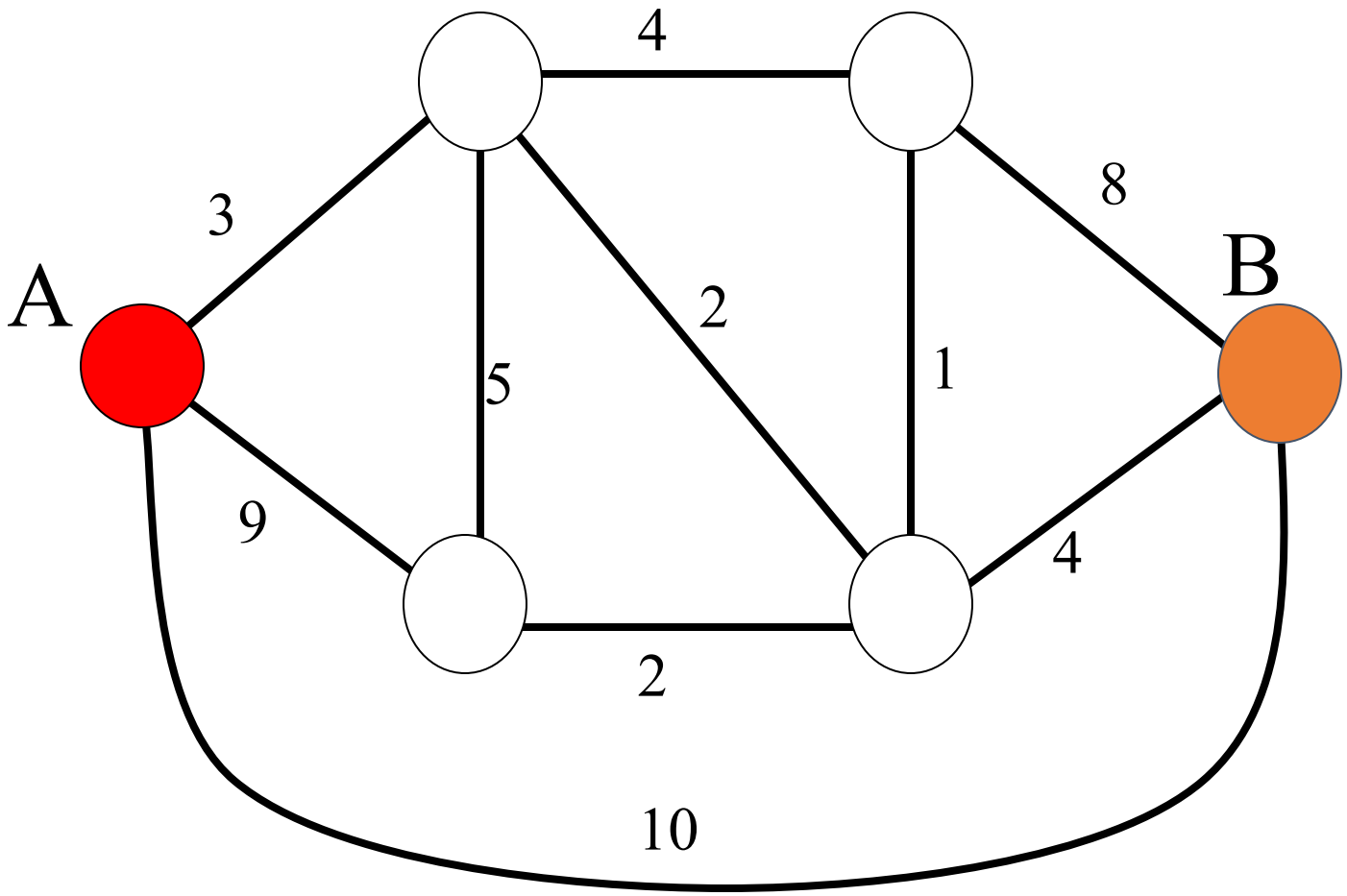
$$L_k \Delta S_k = \Delta T_k$$

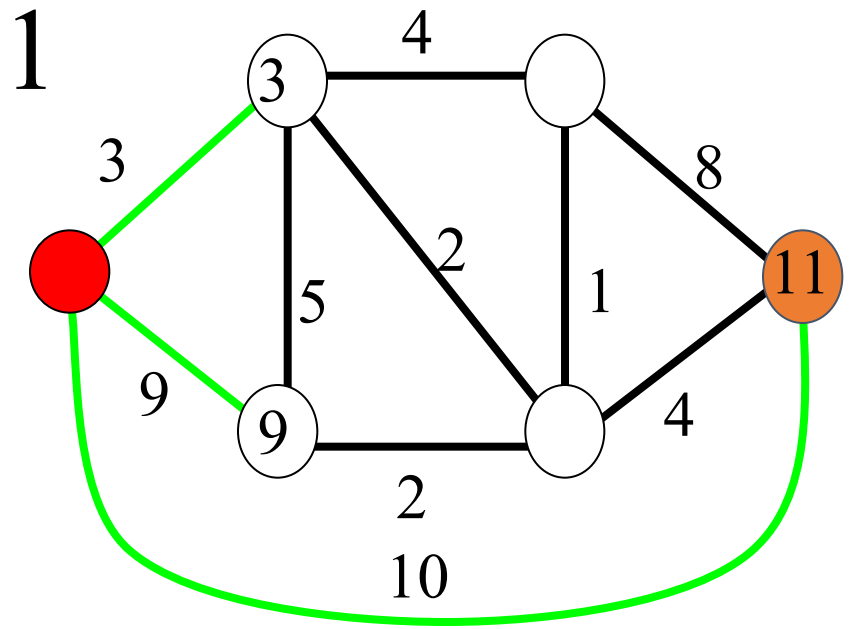
$$S_{k+1} = S_k + \Delta S_k$$

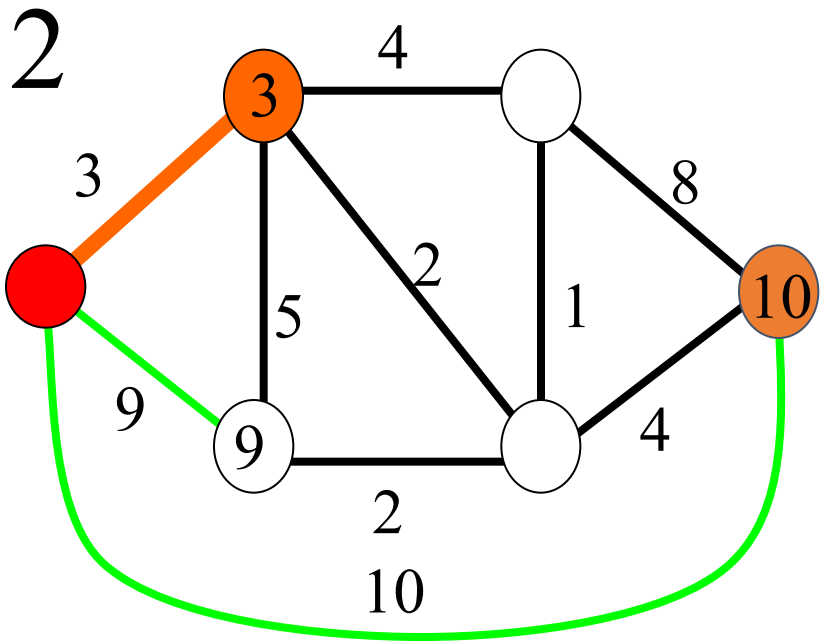
Calculating traveltimes by computer (Raytracing)



Find the shortest path from A to B




1
3
4
3
8
2
1
11
5
9
4
2
10


2
3
4
3
8
2
1
10
5
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2
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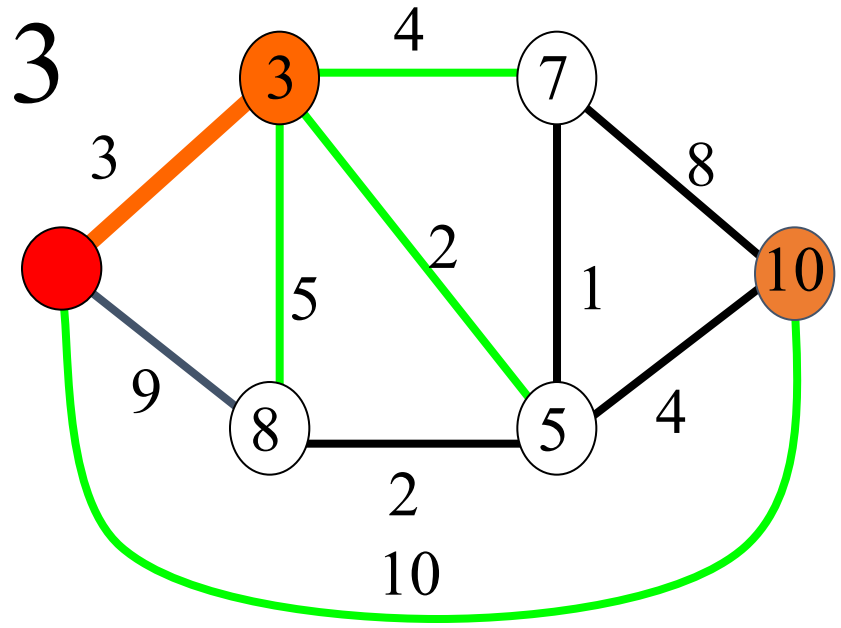
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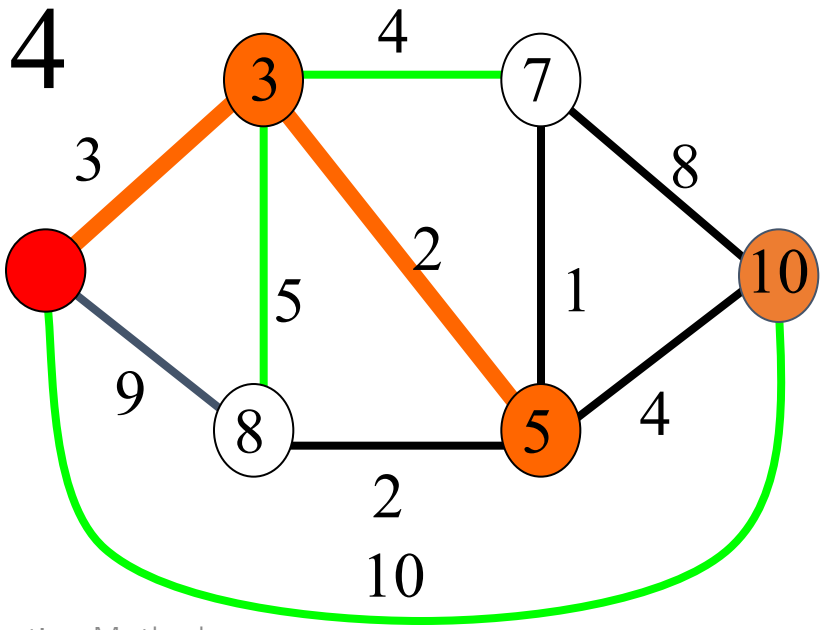
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2
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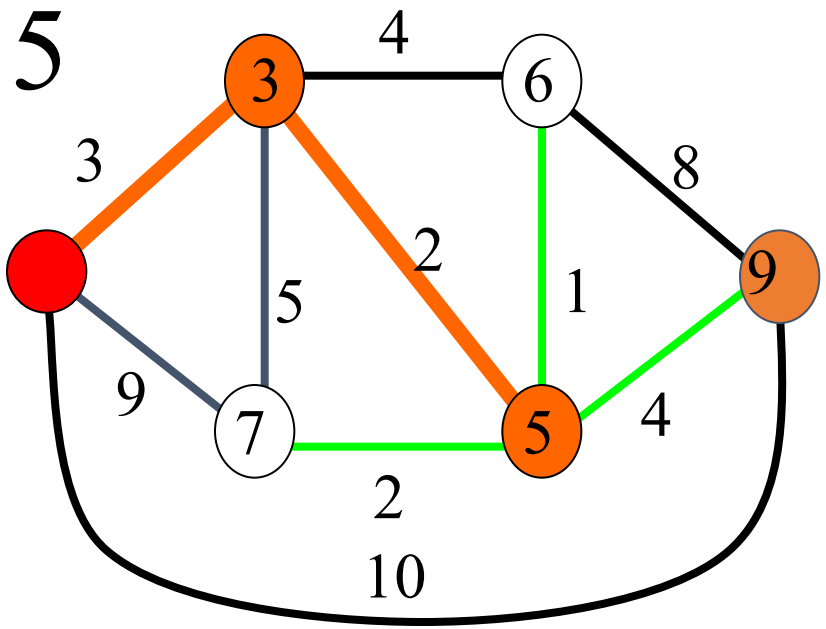
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2
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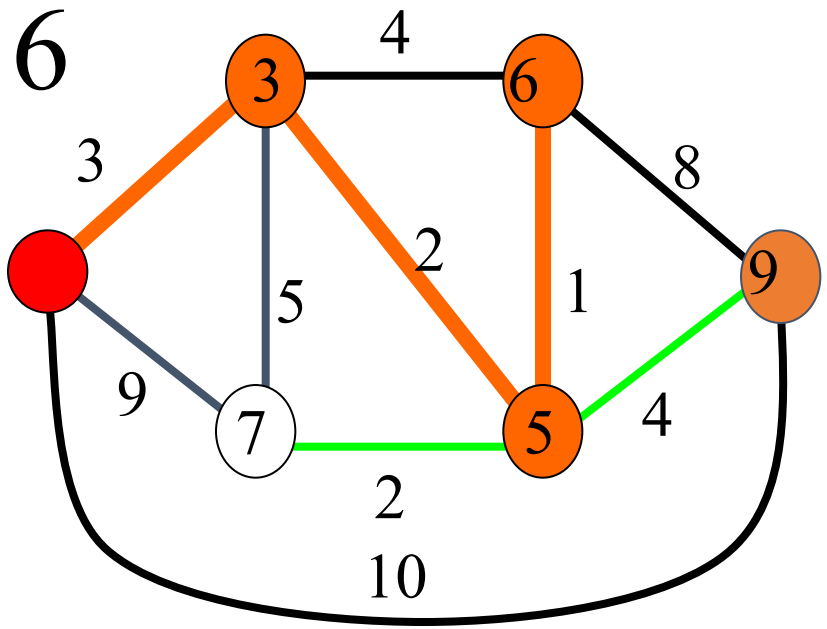
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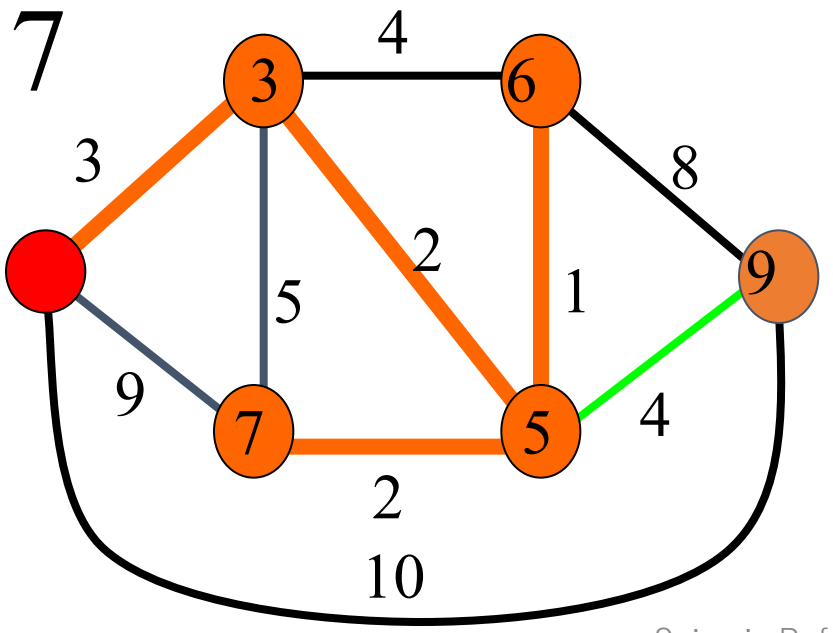
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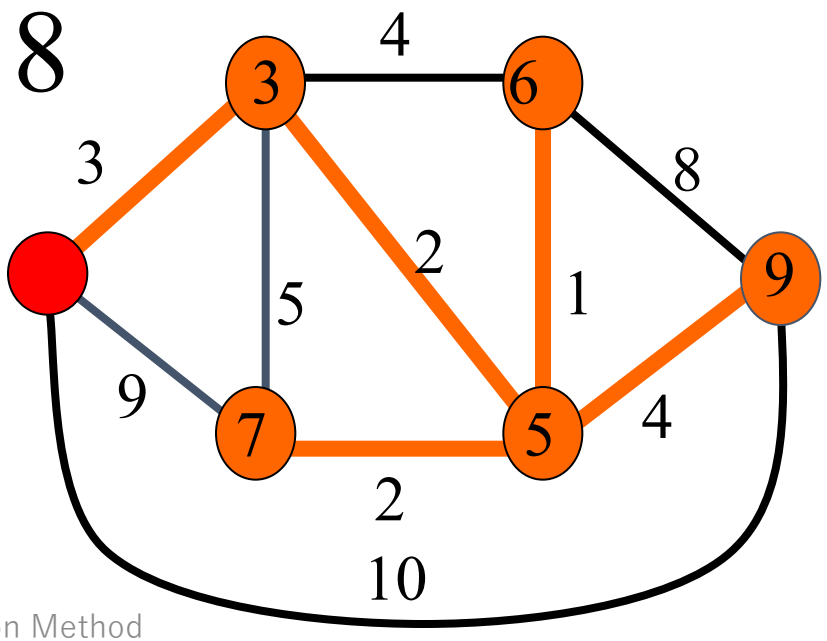
8
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6
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8
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1
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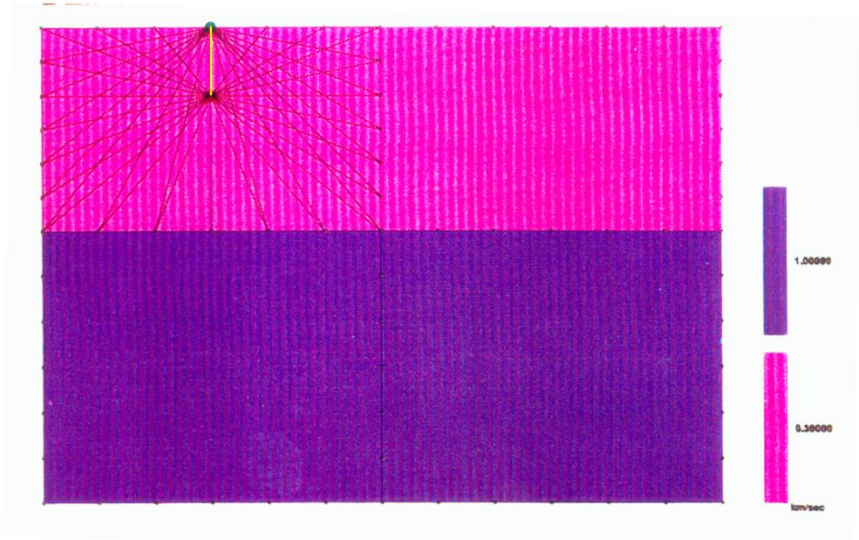
7
5
4
2
10

Example of raytracing (1)

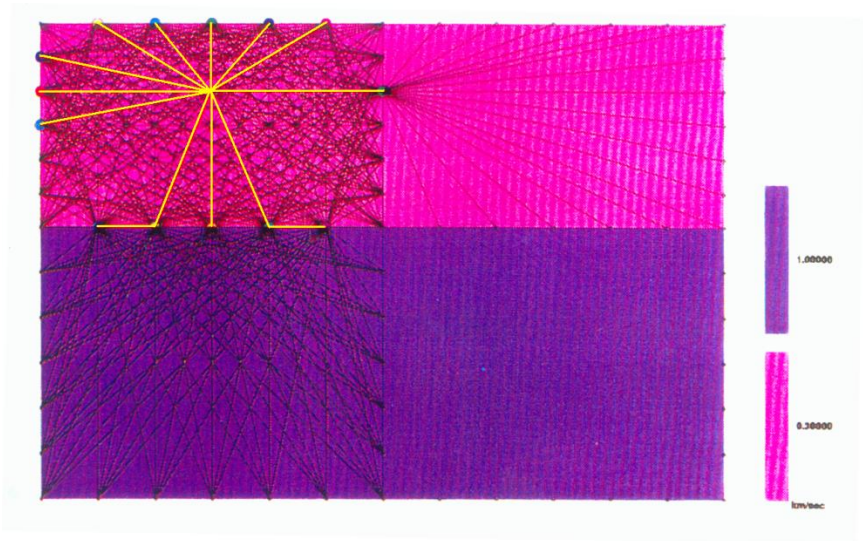
1

2



3

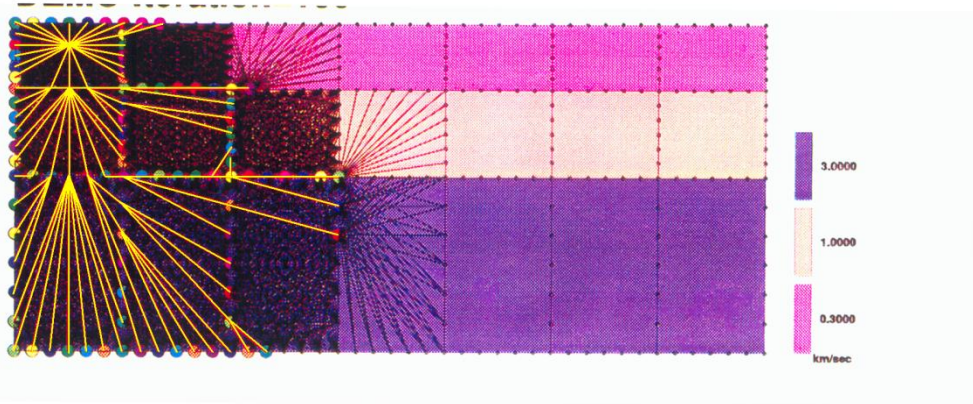
4



Example of raytracing(2)

5

6



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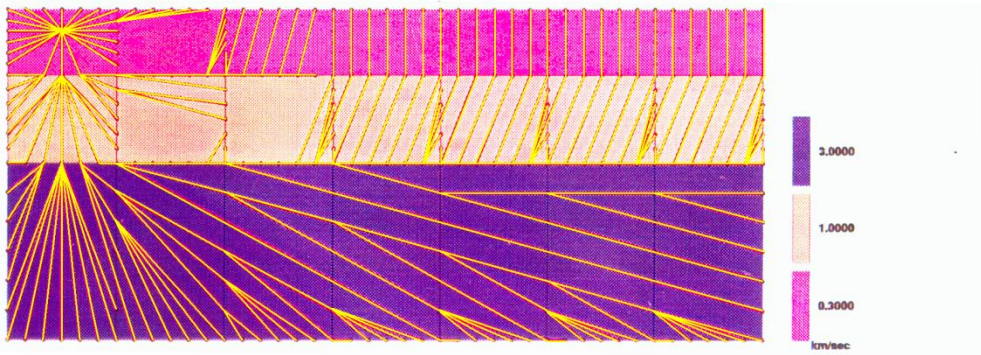
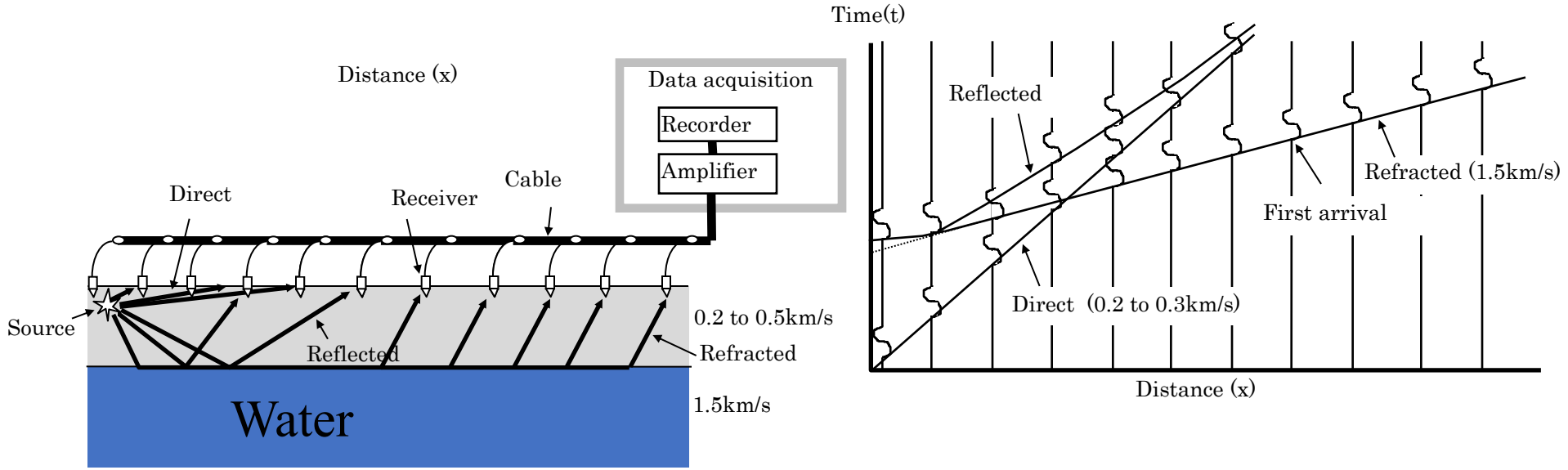


Application of Seismic Refraction Method to Engineering Problems

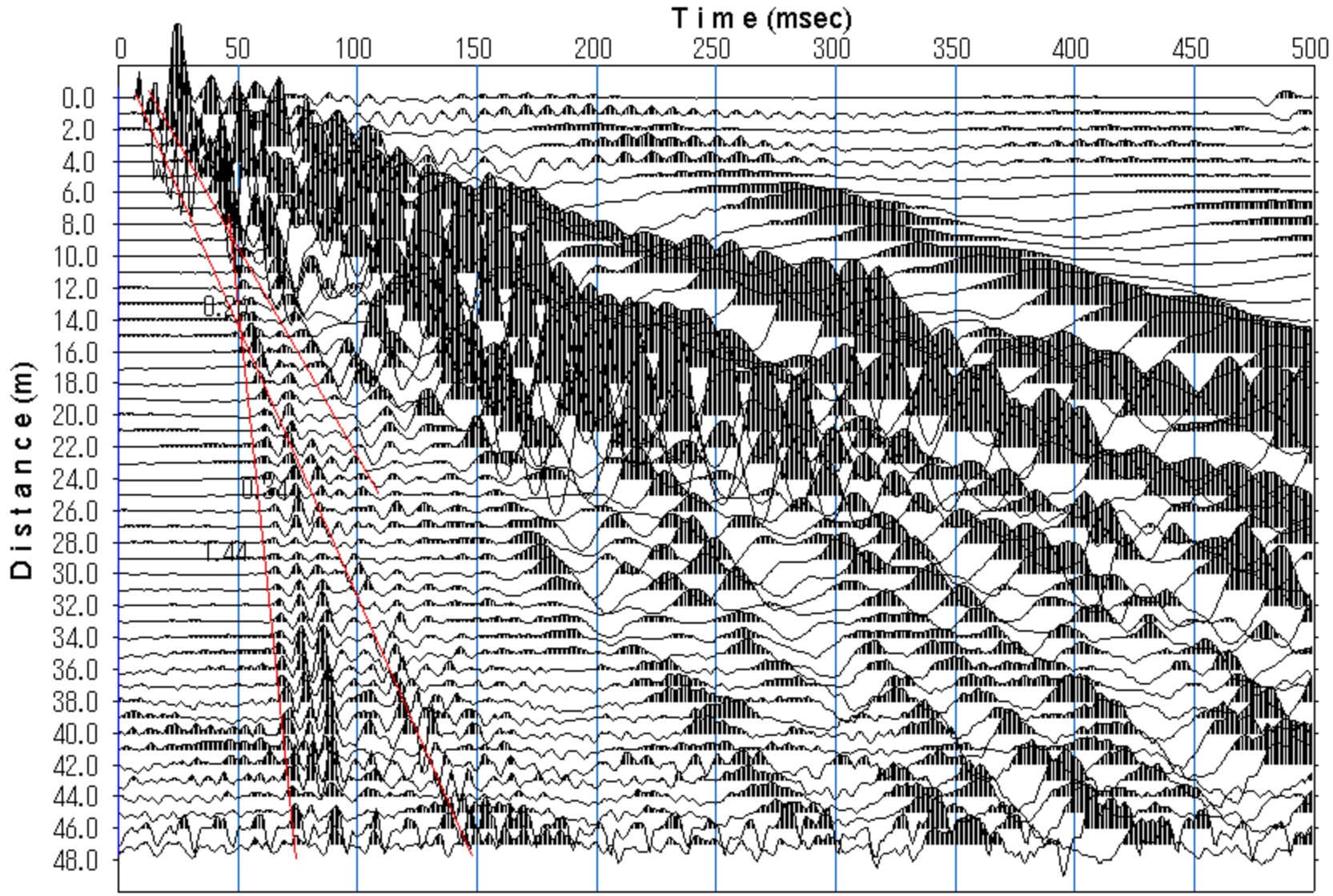
- Ground water
- Slope stability and landslide protections
- Dam constructions

Ground water survey

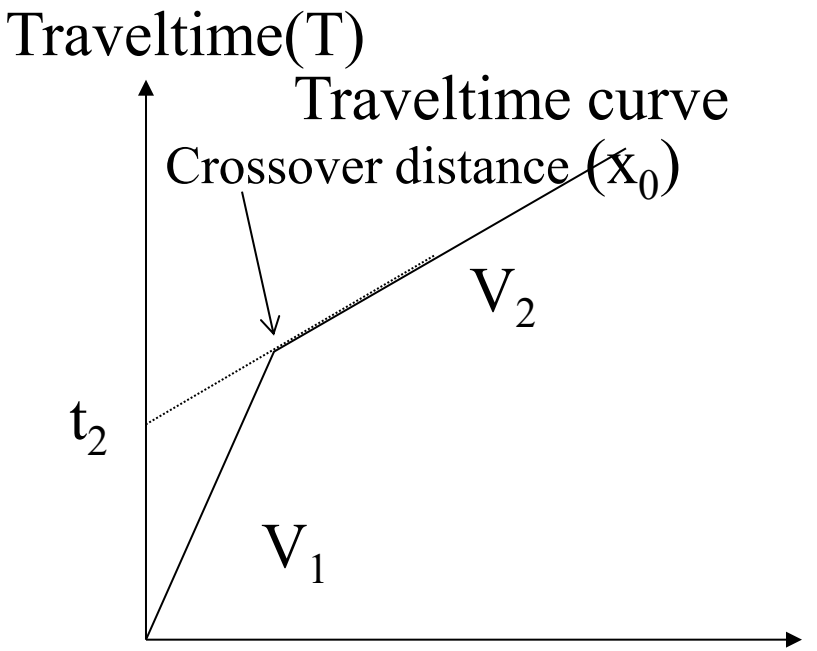
- Velocity of water = 1.5km/s.
- Weathered or unconsolidated layer < 1.5km/s (0.2 to 0.5km/s).
- Air = 0.3km/s



Ground water survey



Ground water survey



t_2=Intercept time

Crossover distance : x_0

$$T_1 = T_2$$

$$T_2 = \frac{2h\cos i_c}{V_1} + \frac{x_0}{V_2} = \frac{x_0}{V_1} = T_1$$

$$x_0 = 2h\sqrt{\frac{V_2 + V_1}{V_2 - V_1}} = \text{Crossover distance}$$

$$h = \frac{x_0}{2}\sqrt{\frac{V_2 - V_1}{V_2 + V_1}} = \text{Depth}$$

Ground water survey

$$h = \frac{x_0}{2}\sqrt{\frac{V_2 - V_1}{V_2 + V_1}} = \text{Depth}$$

x_0=10m

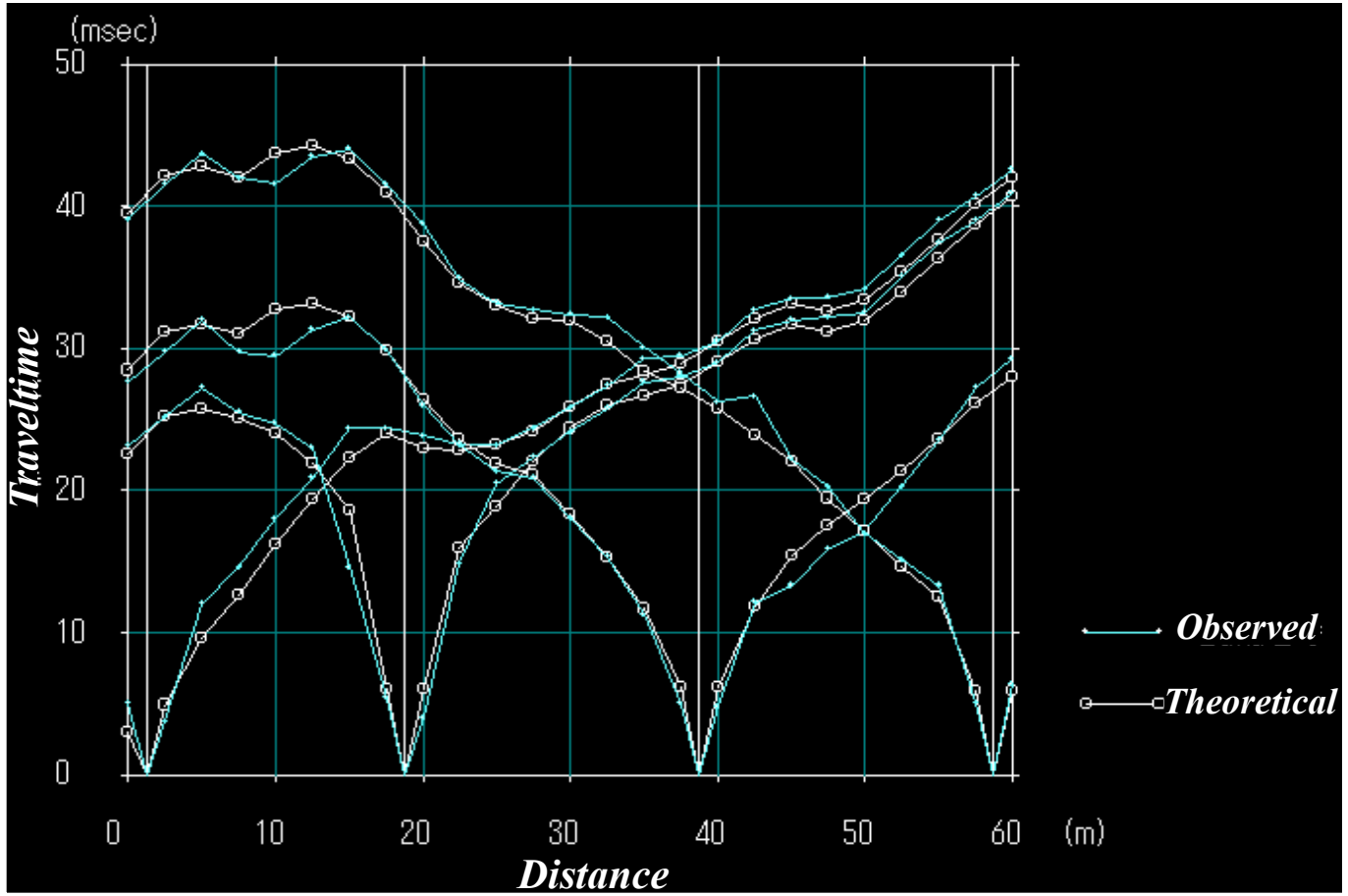
V1=260m/s

V2=1440m/s

Depth $$h = \frac{10}{2}\sqrt{\frac{1440 - 260}{1440 + 260}} = 4.7 \text{ m}$$

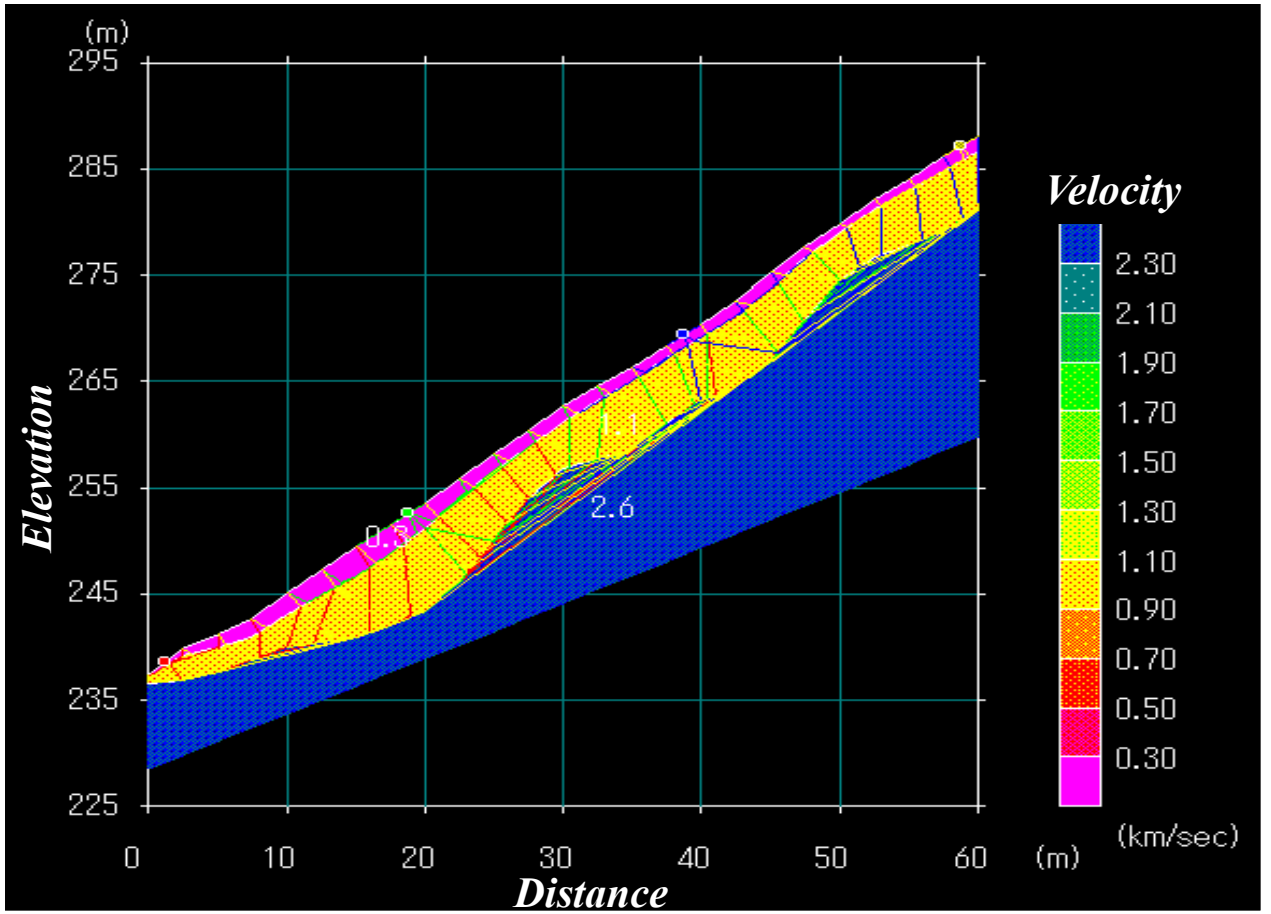
Slope Stability

Data (traveltime curves)



Slope Stability

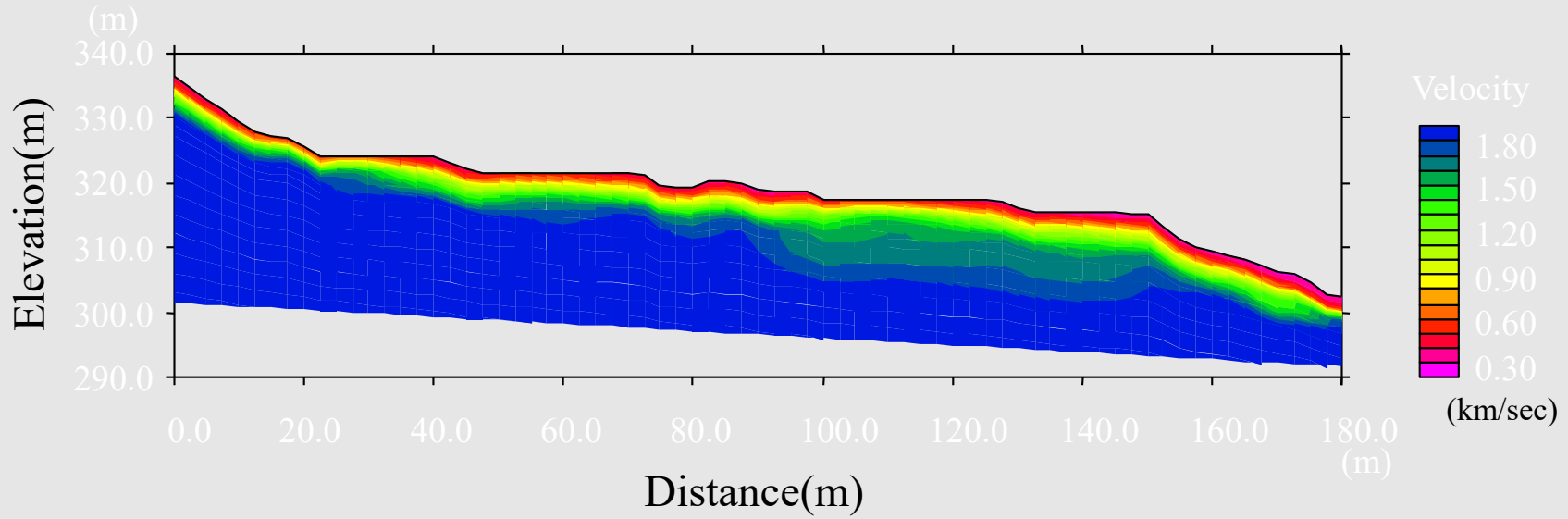
Velocity model



Application to slope stability

Landslide investigation

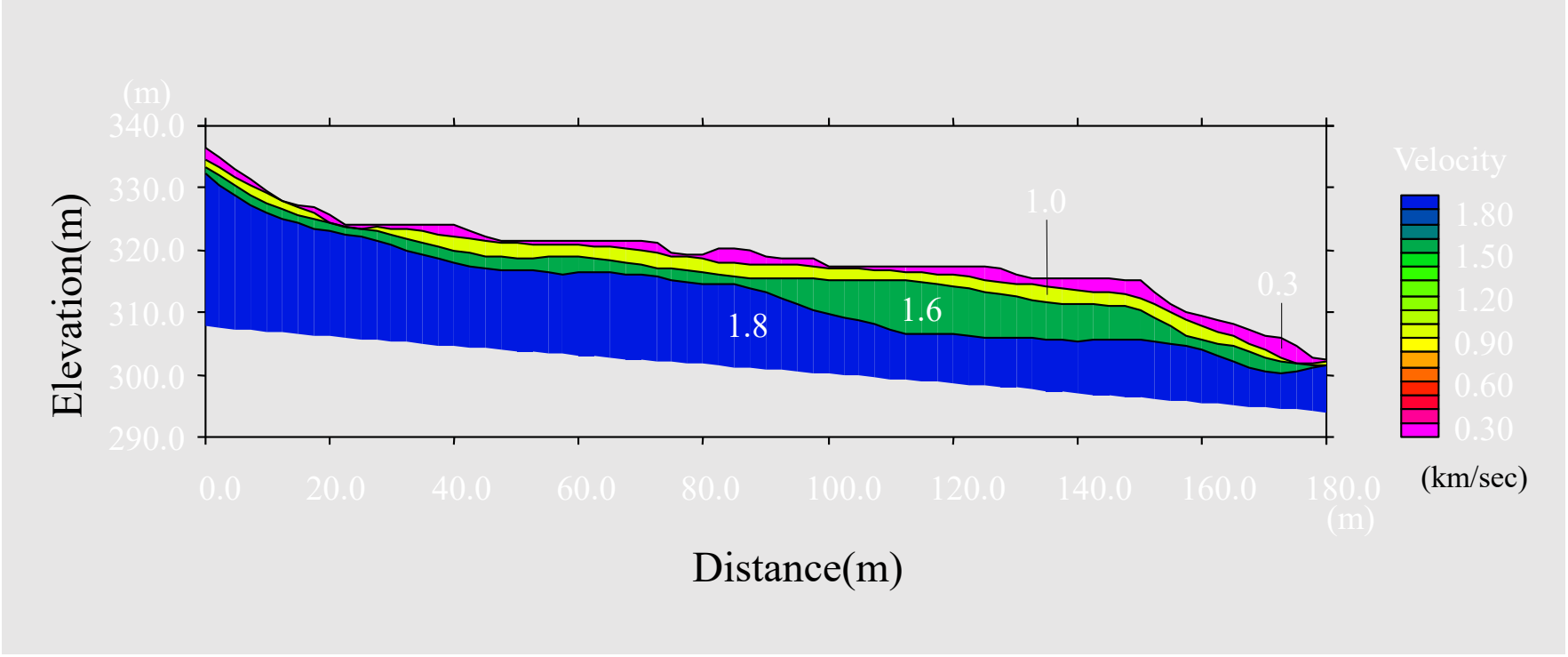
Smoothed velocity model obtained
by traveltime tomography



Application to slope stability

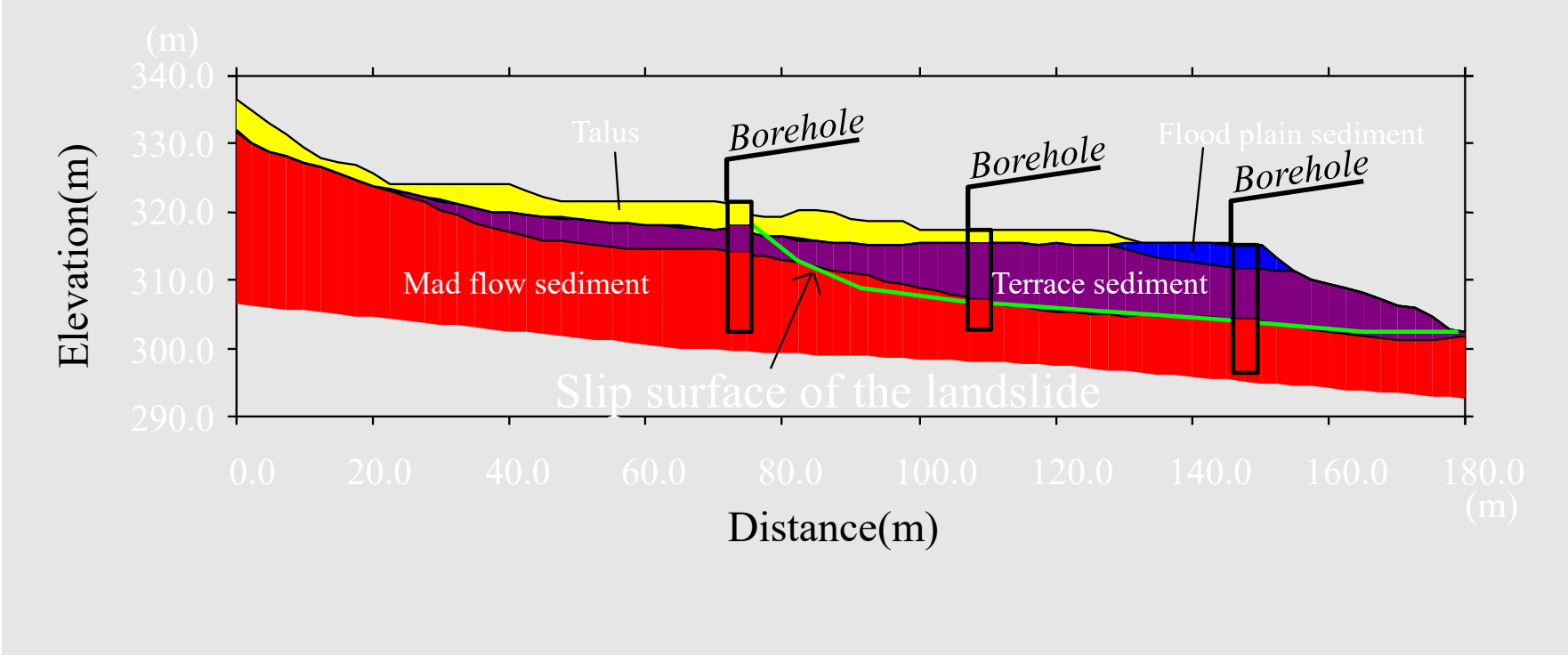
Landslide investigation

Layered velocity model converted
from smoothed model



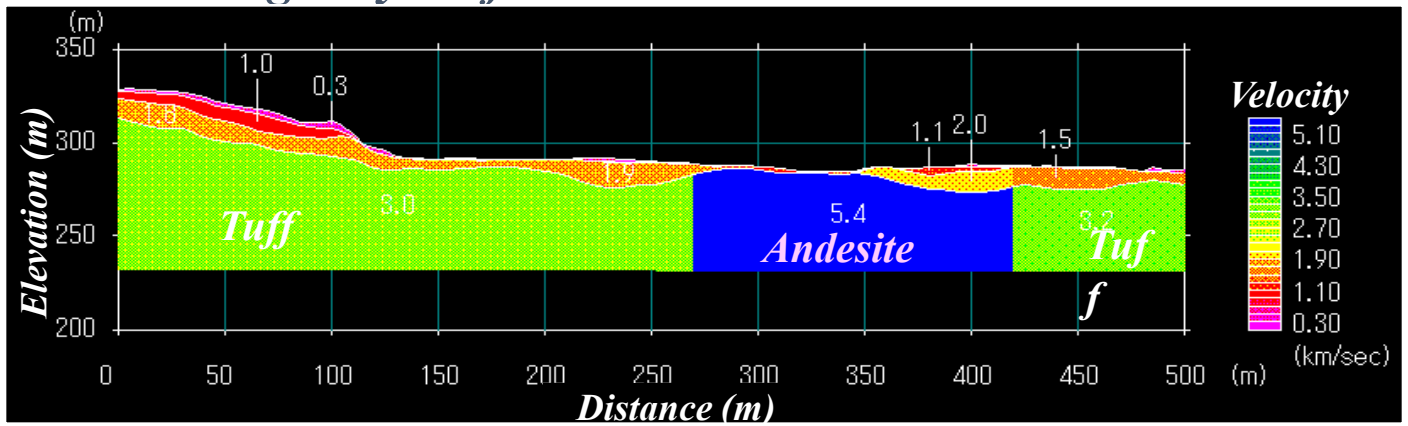
Application to slope stability
Landslide investigation

Geological profile and borehole information at the exploration site

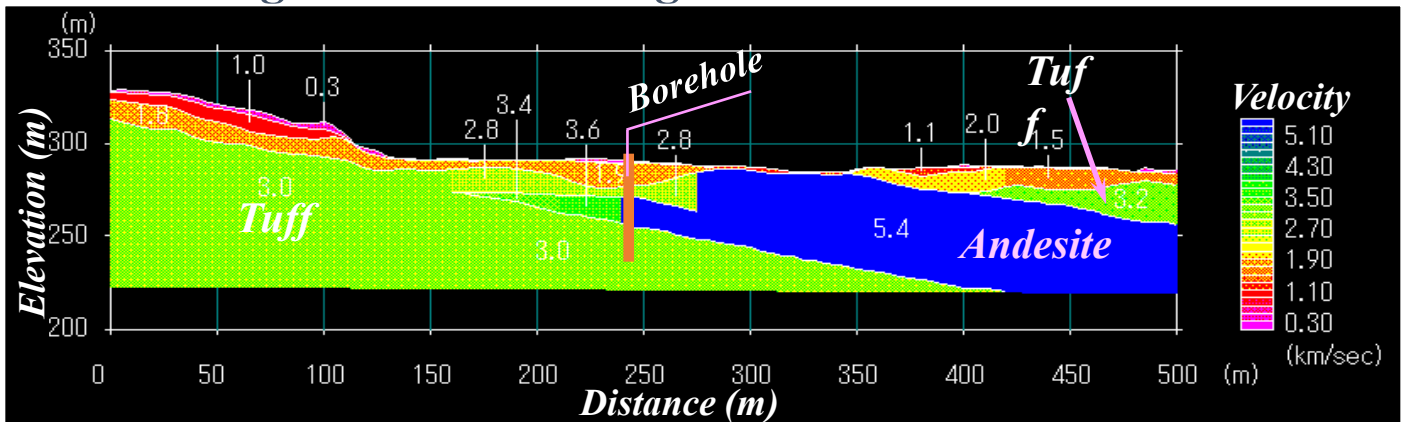


Application example for dam site

Result using only surface data



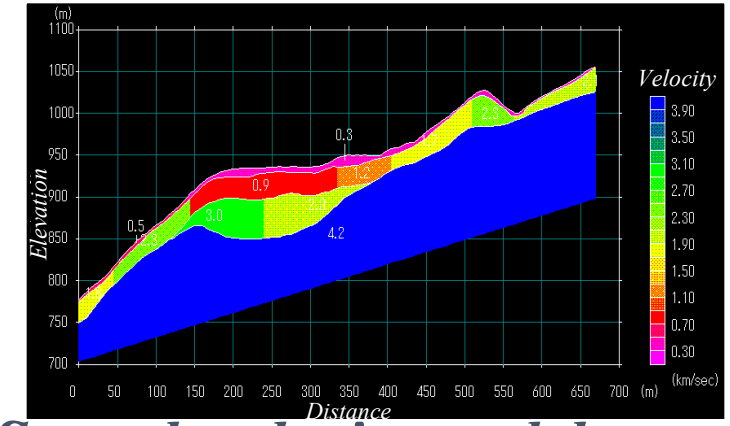
Result using borehole data together



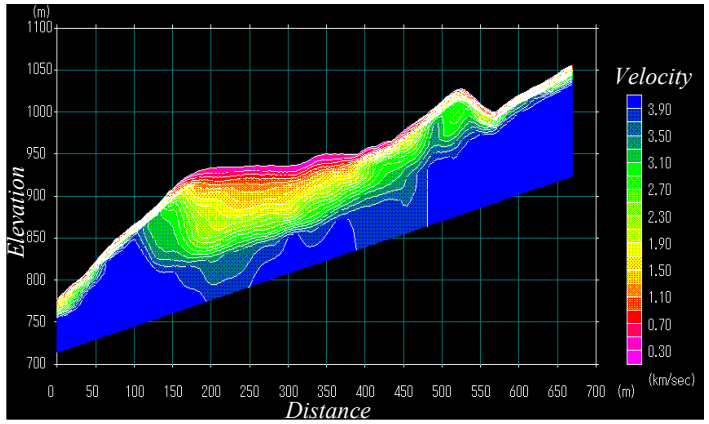
Application for dam site

Quarry investigation

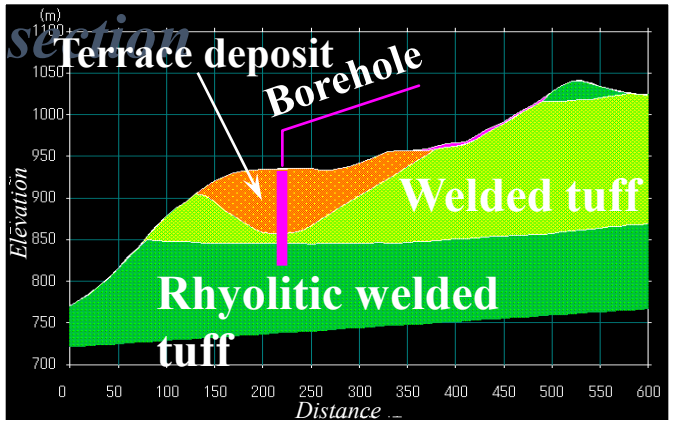
Layered velocity model



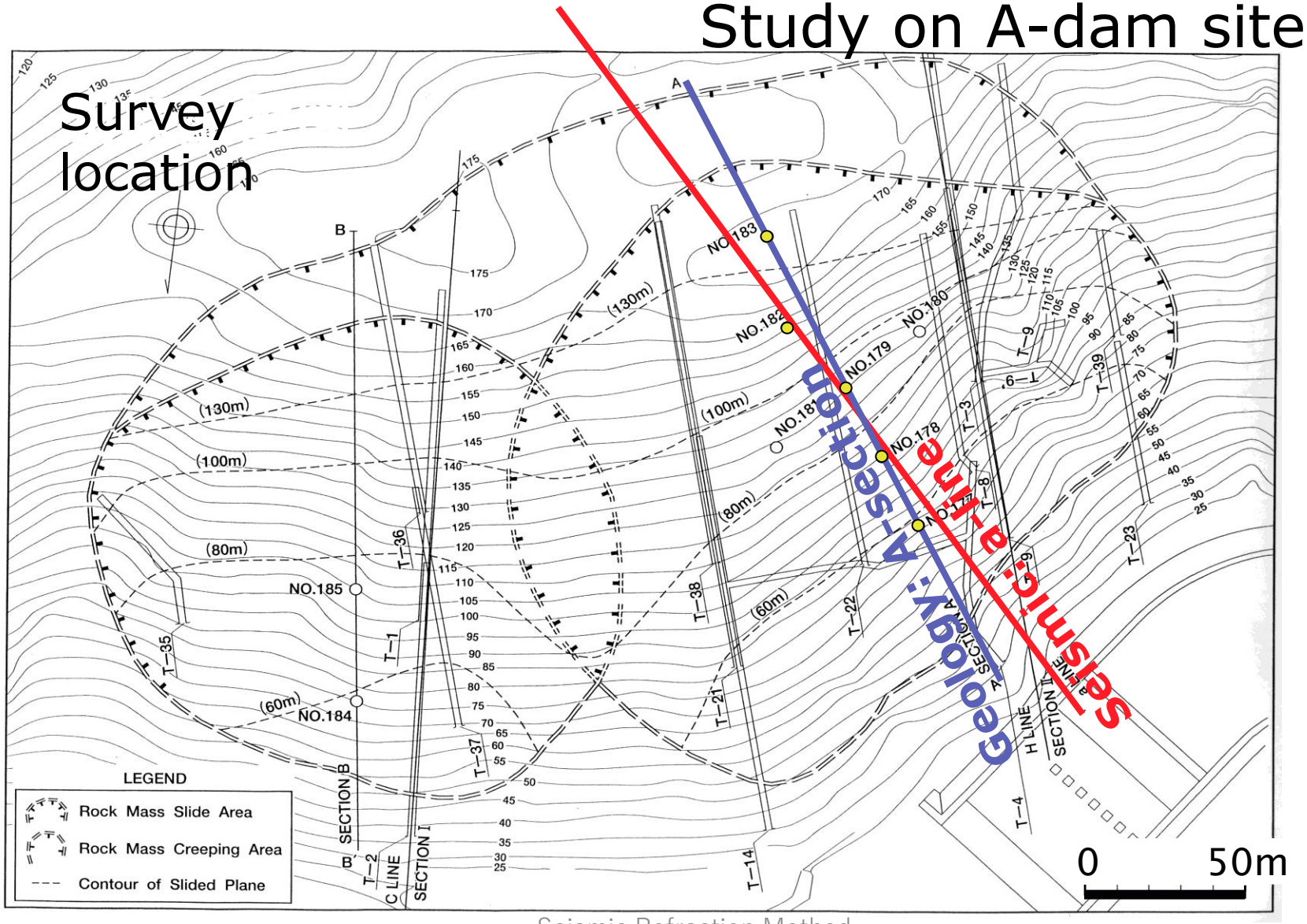
Smooth velocity model



Interpreted geological section



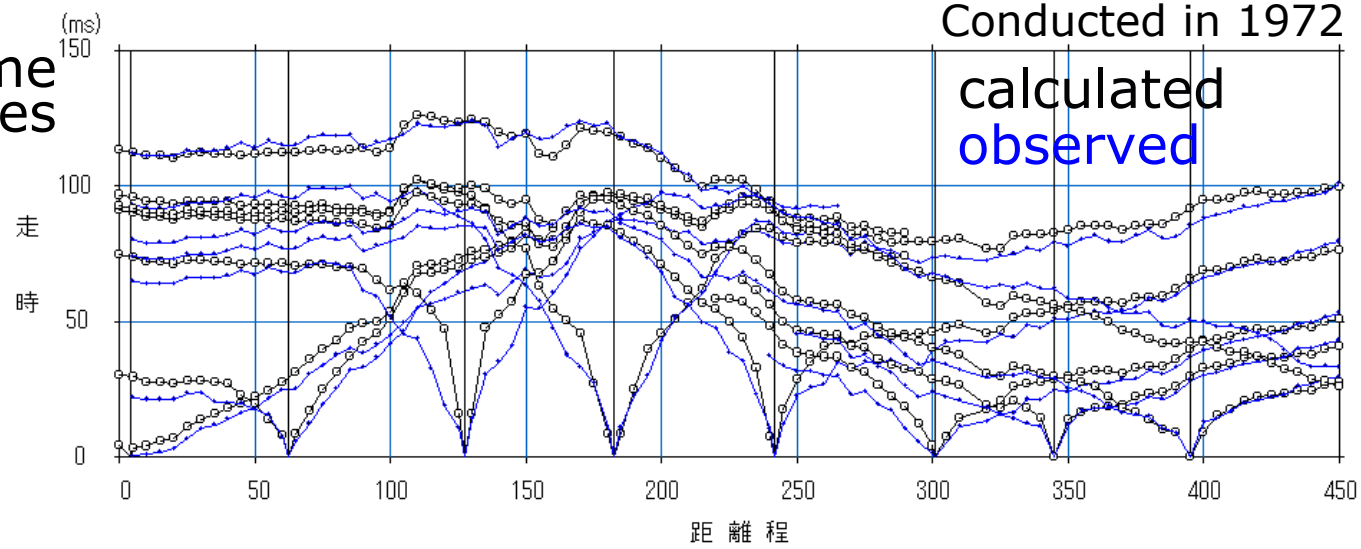
Land slide in the dam site



Conventional seismic refraction prospecting; A-dam/a-line

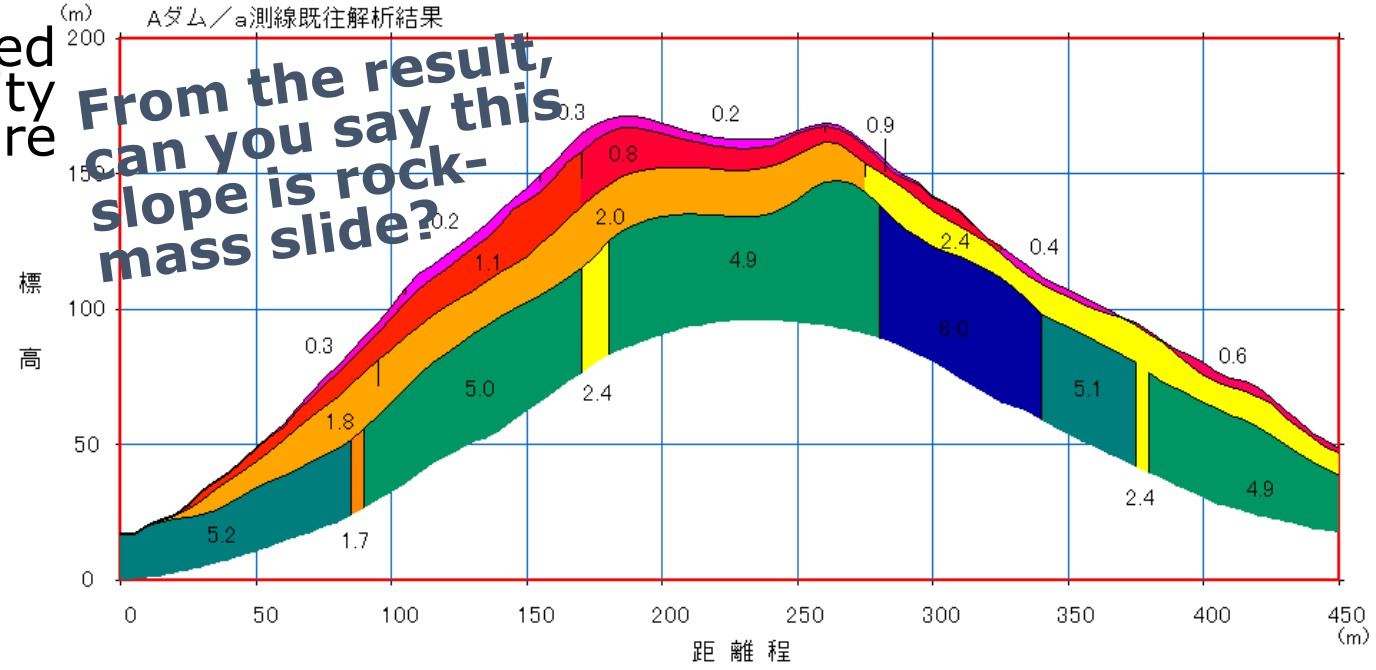
Traveltime curves

Conducted in 1972



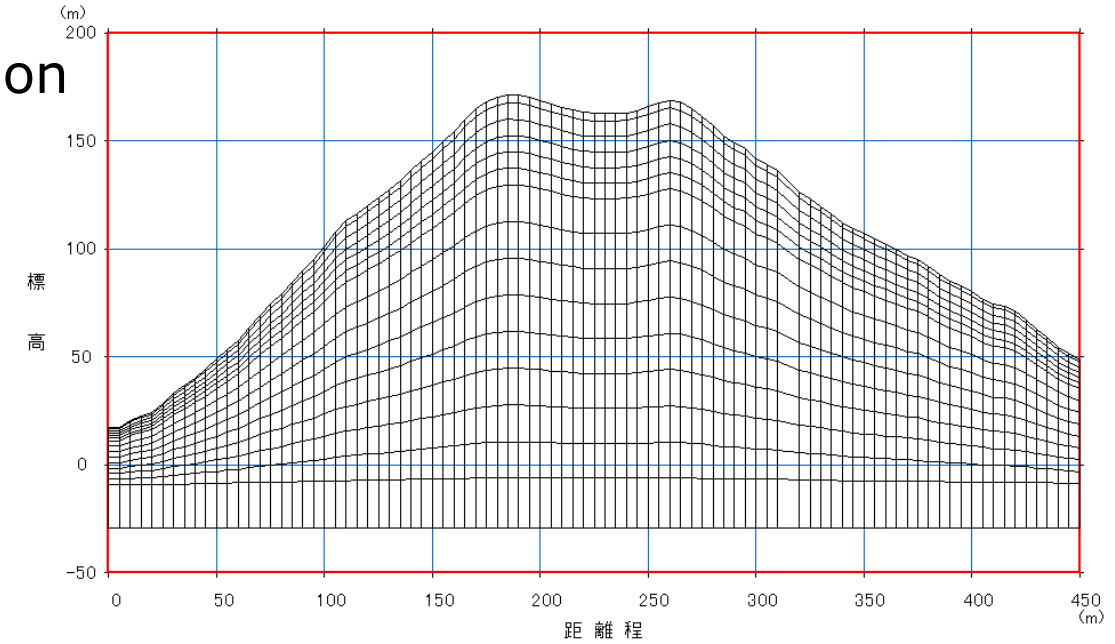
Analyzed velocity structure

From the result, can you say this slope is rock-mass slide?

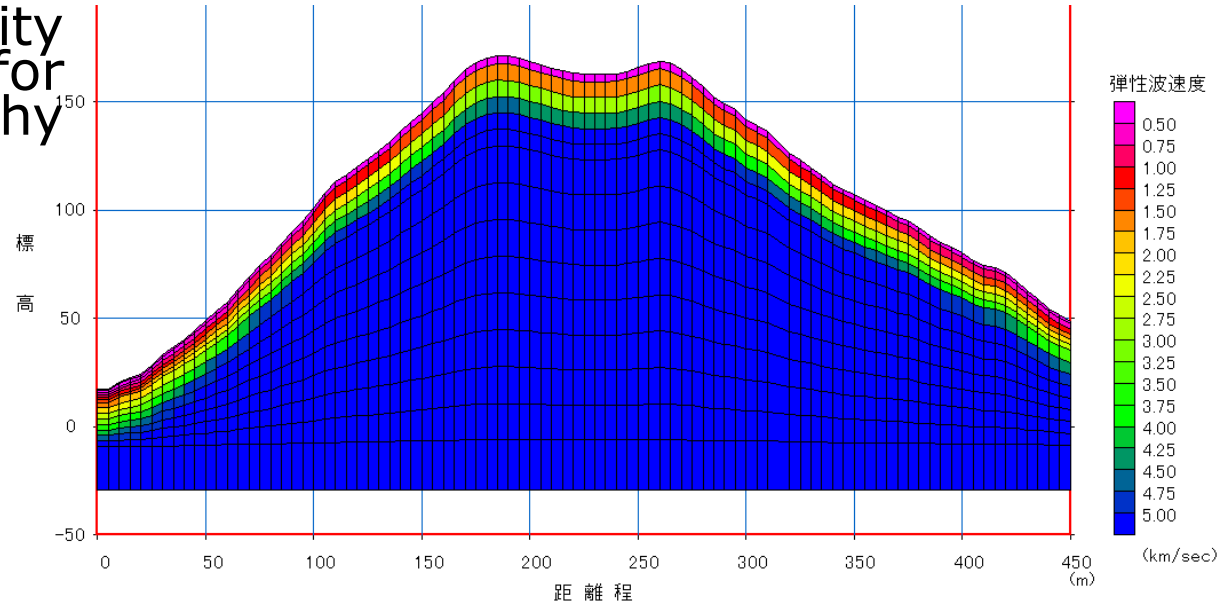


Cell division and initial model for the reanalysis

Cell division

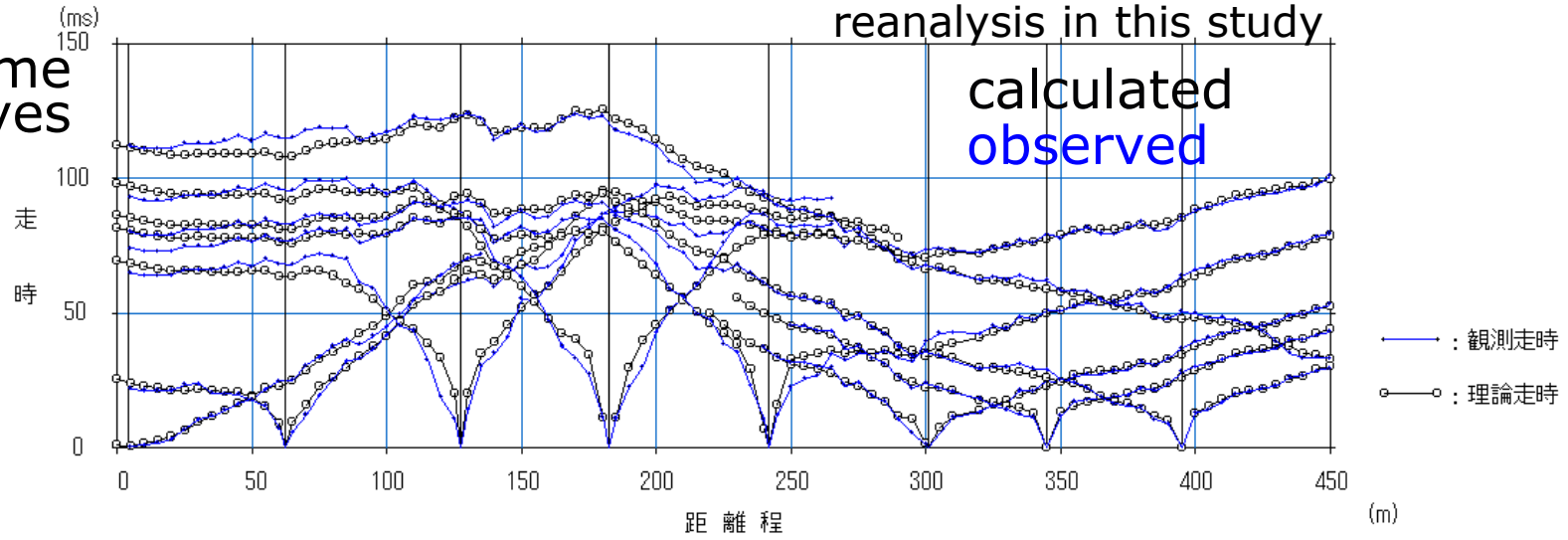


Initial velocity model for tomography

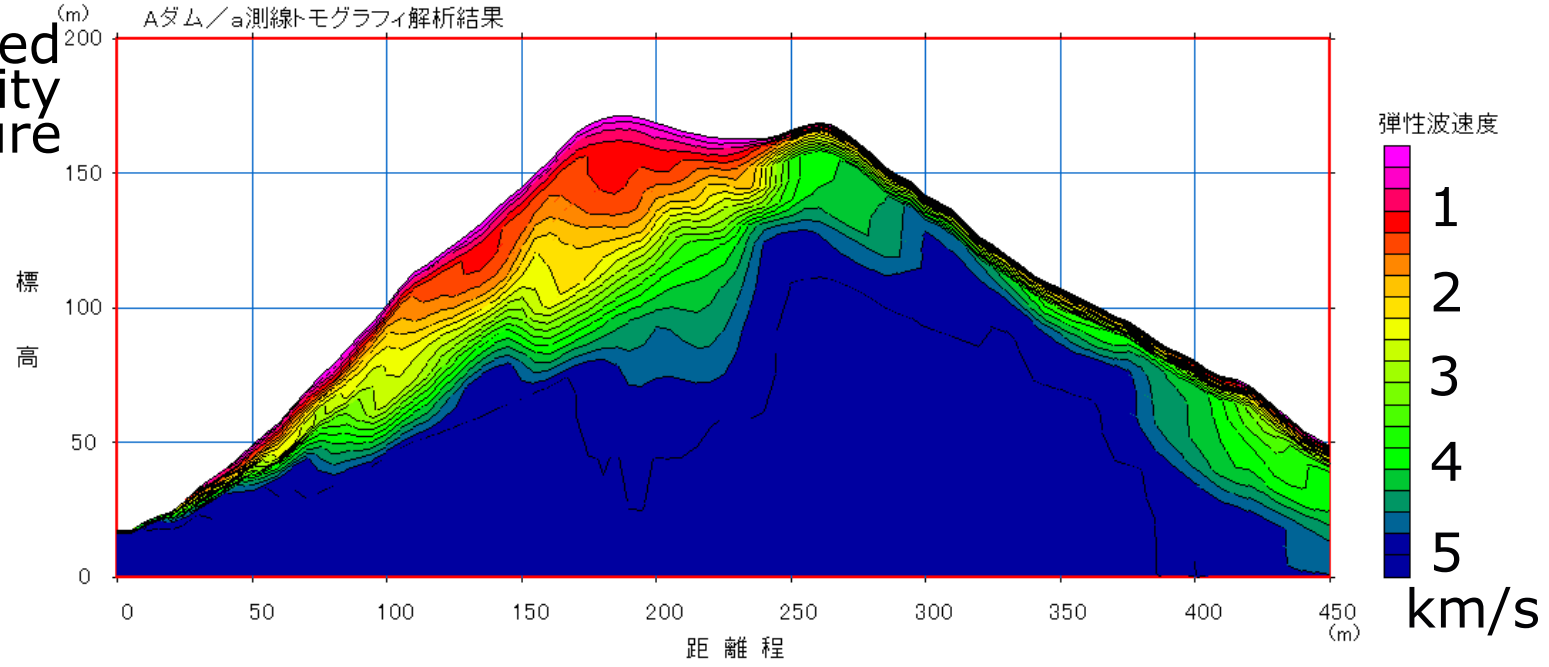


Tomographic reconstruction of refraction data; A-dam/a-line

Traveltime curves



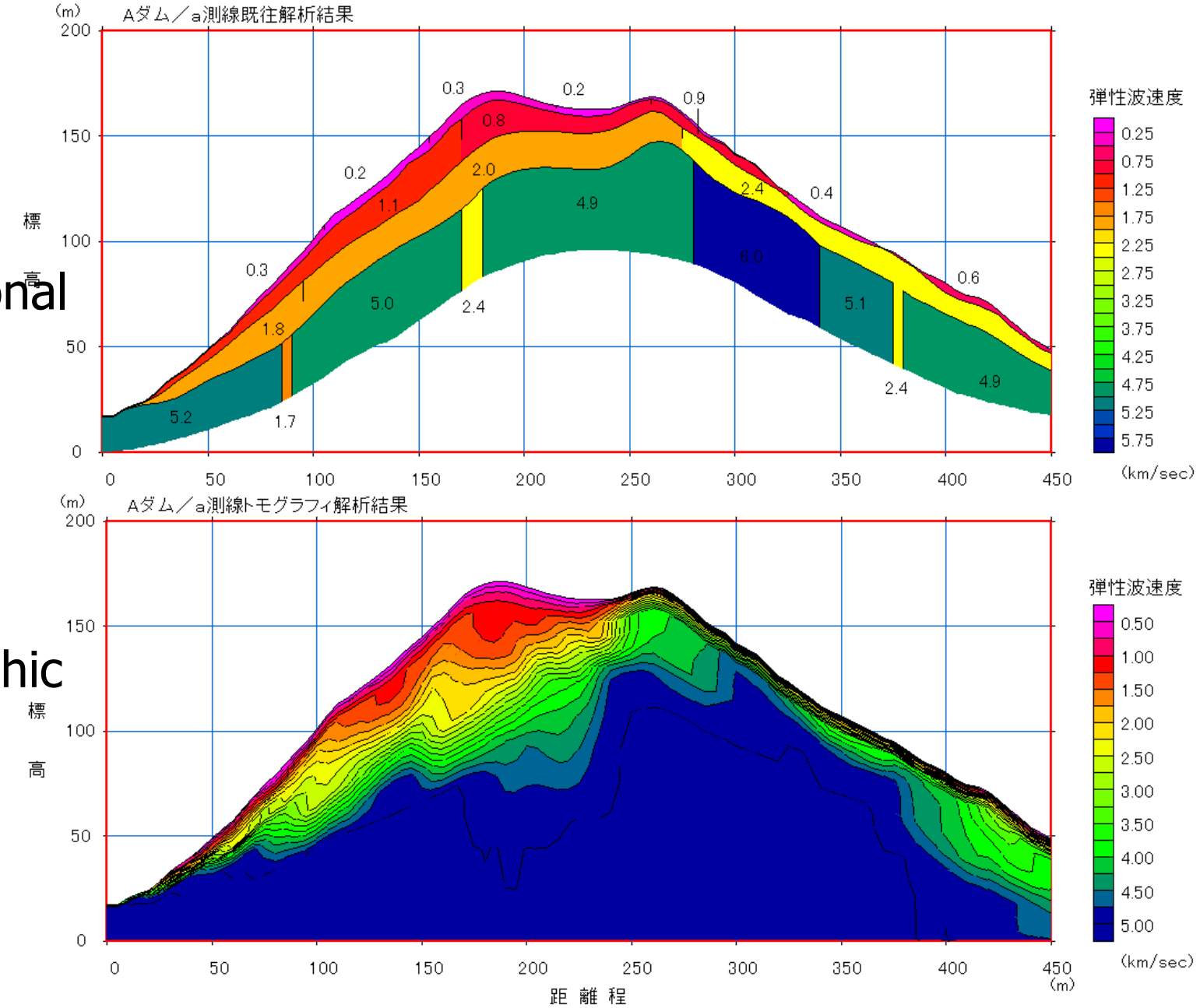
Analyzed velocity structure



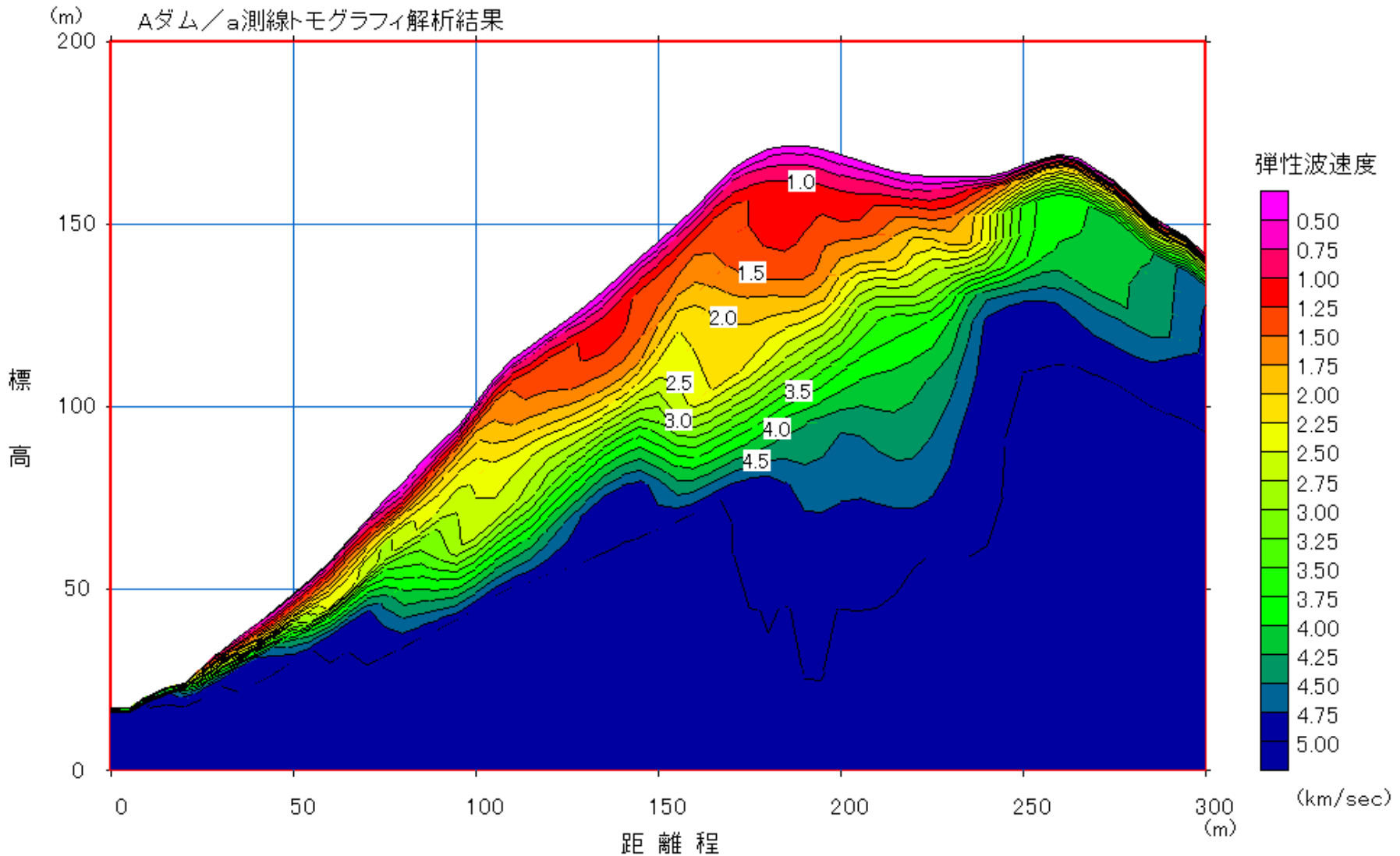
Conventional

vs.

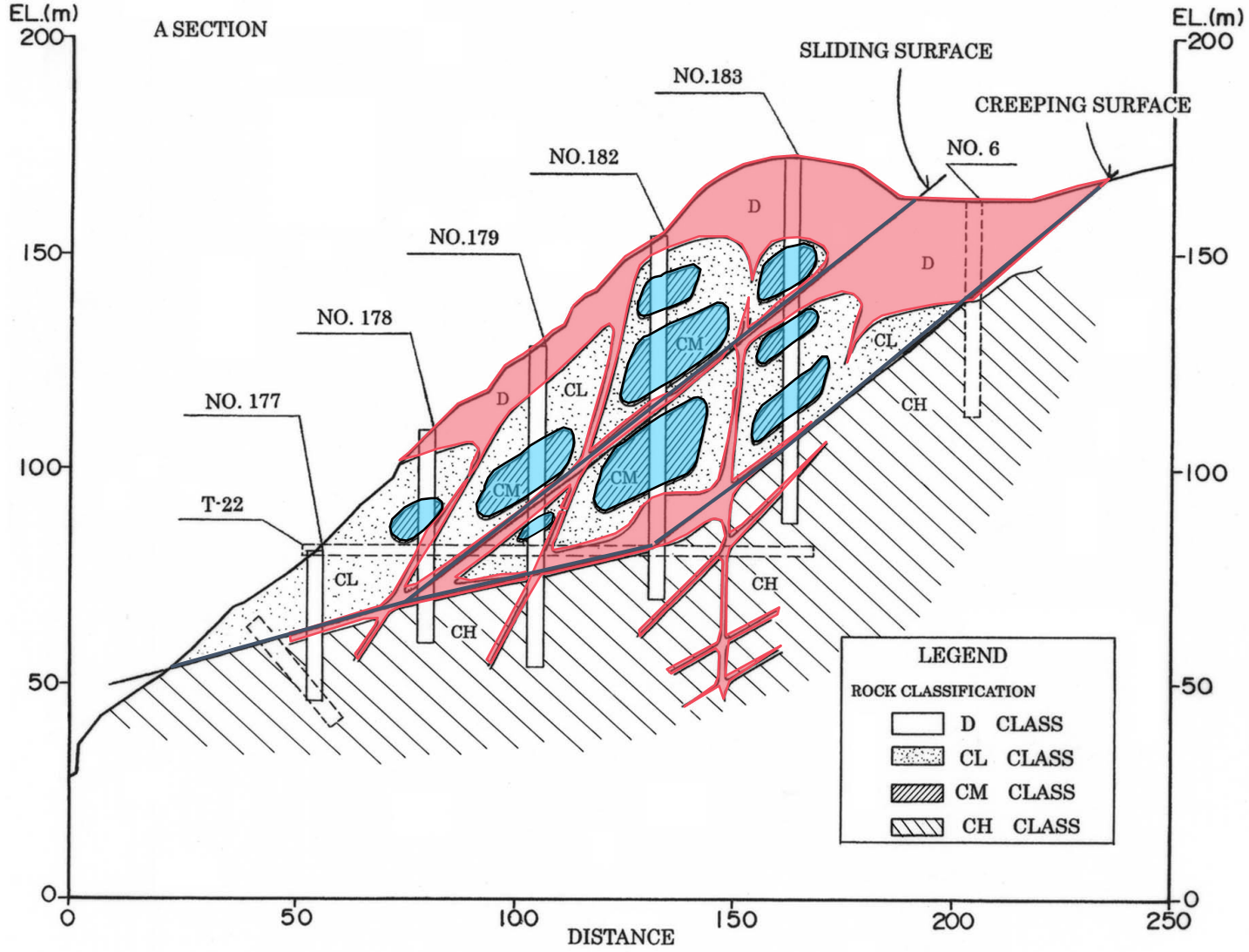
Tomographic



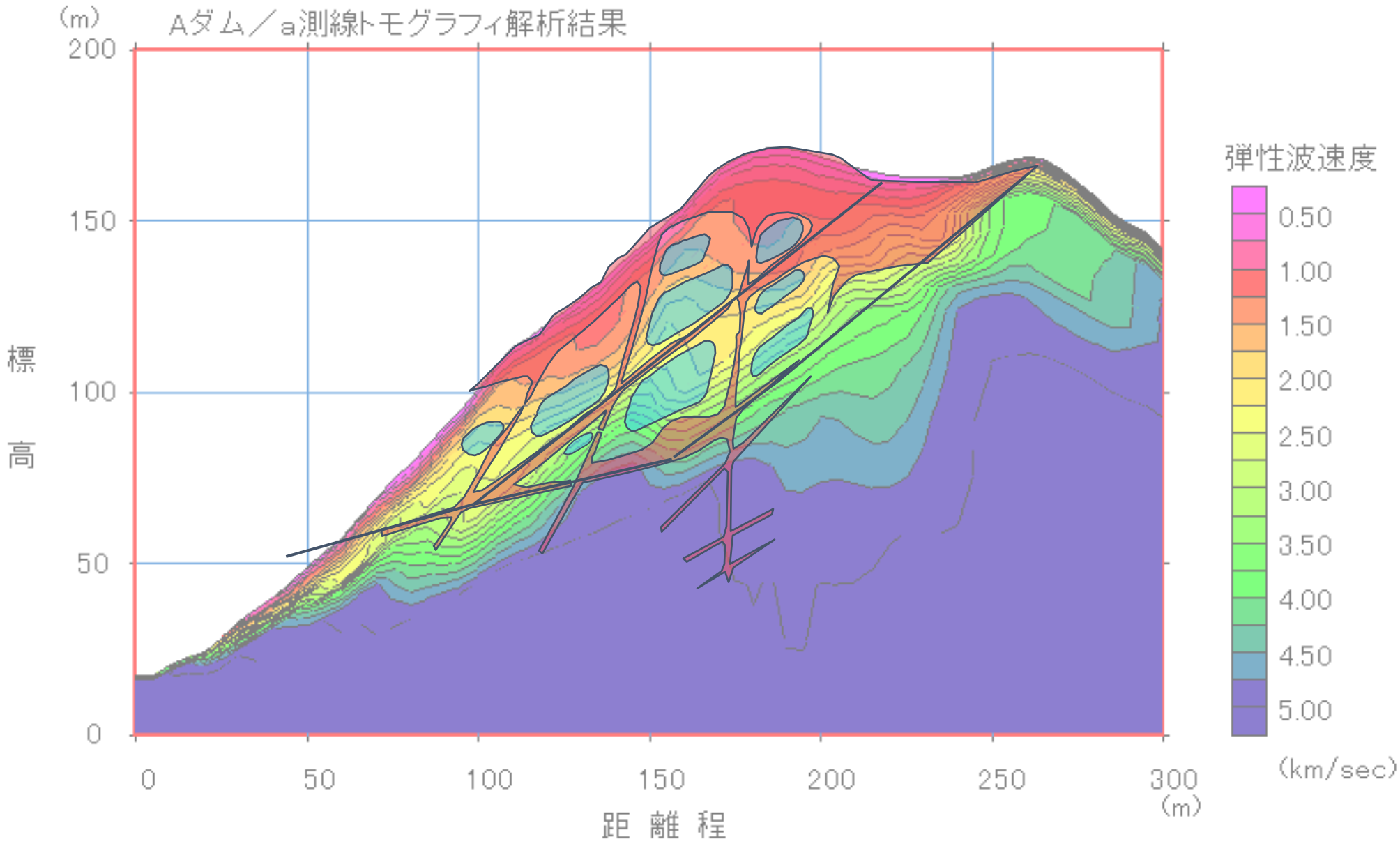
Left Slope


Aダム／a測線トモグラフィ解析結果
(m)
200
150
100
50
0
標
高
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
0
50
100
150
200
250
300
(m)
距 離 程
弾性波速度
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25
2.50
2.75
3.00
3.25
3.50
3.75
4.00
4.25
4.50
4.75
5.00
(km/sec)

Rock Grade Section

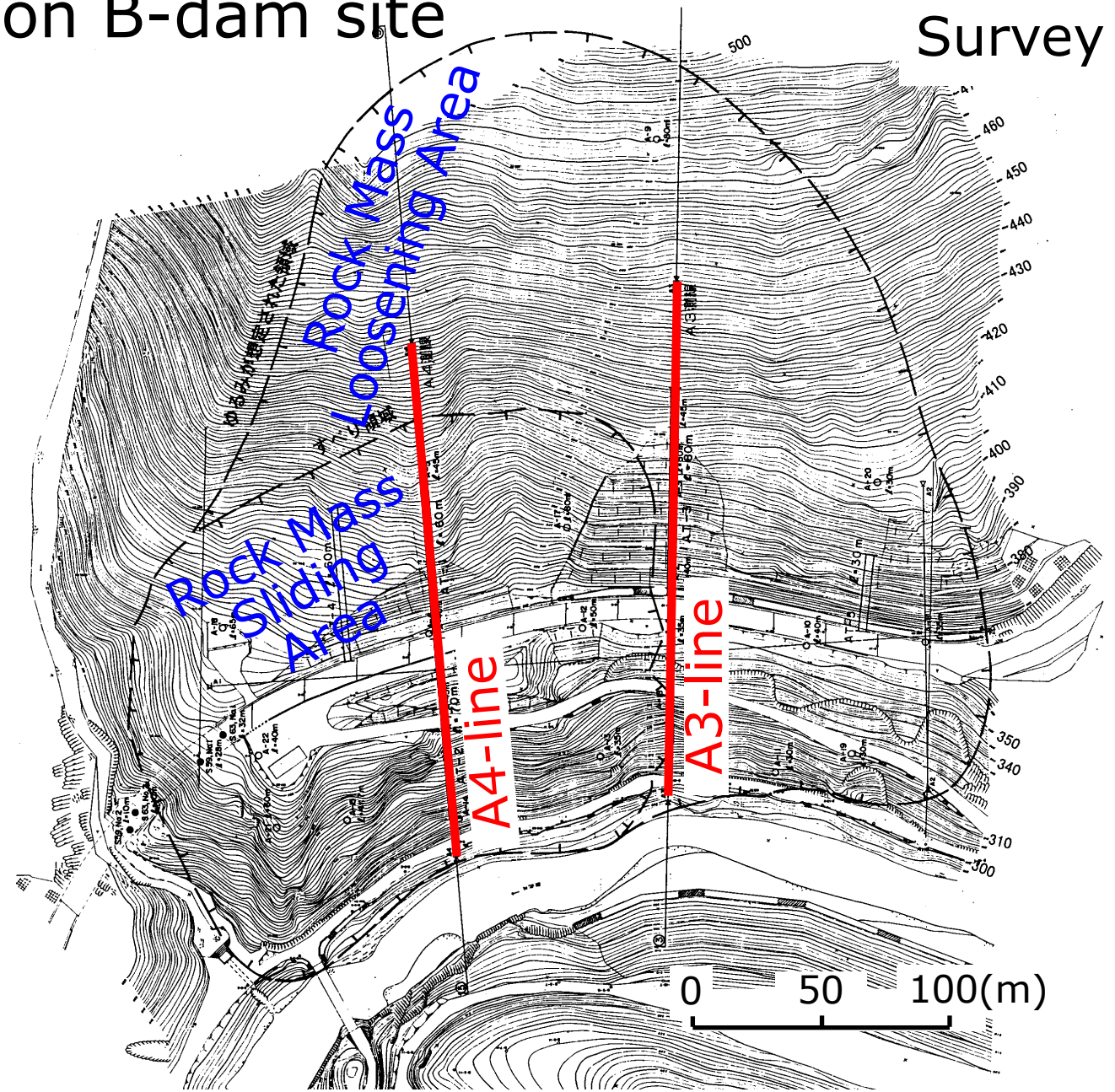


Comparisons between seismic velocity and rock grade

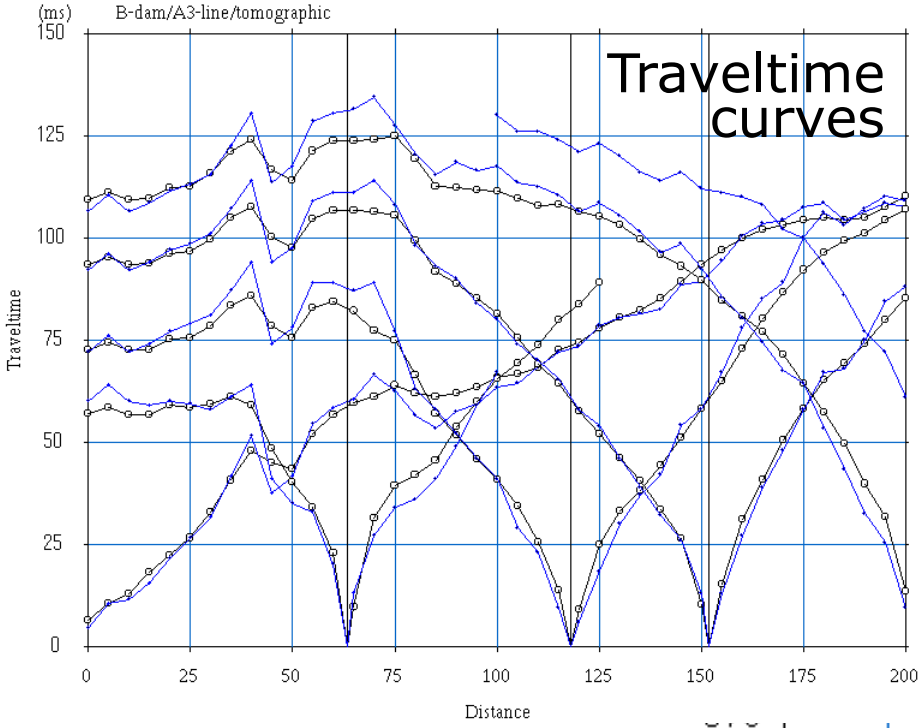


Study on B-dam site

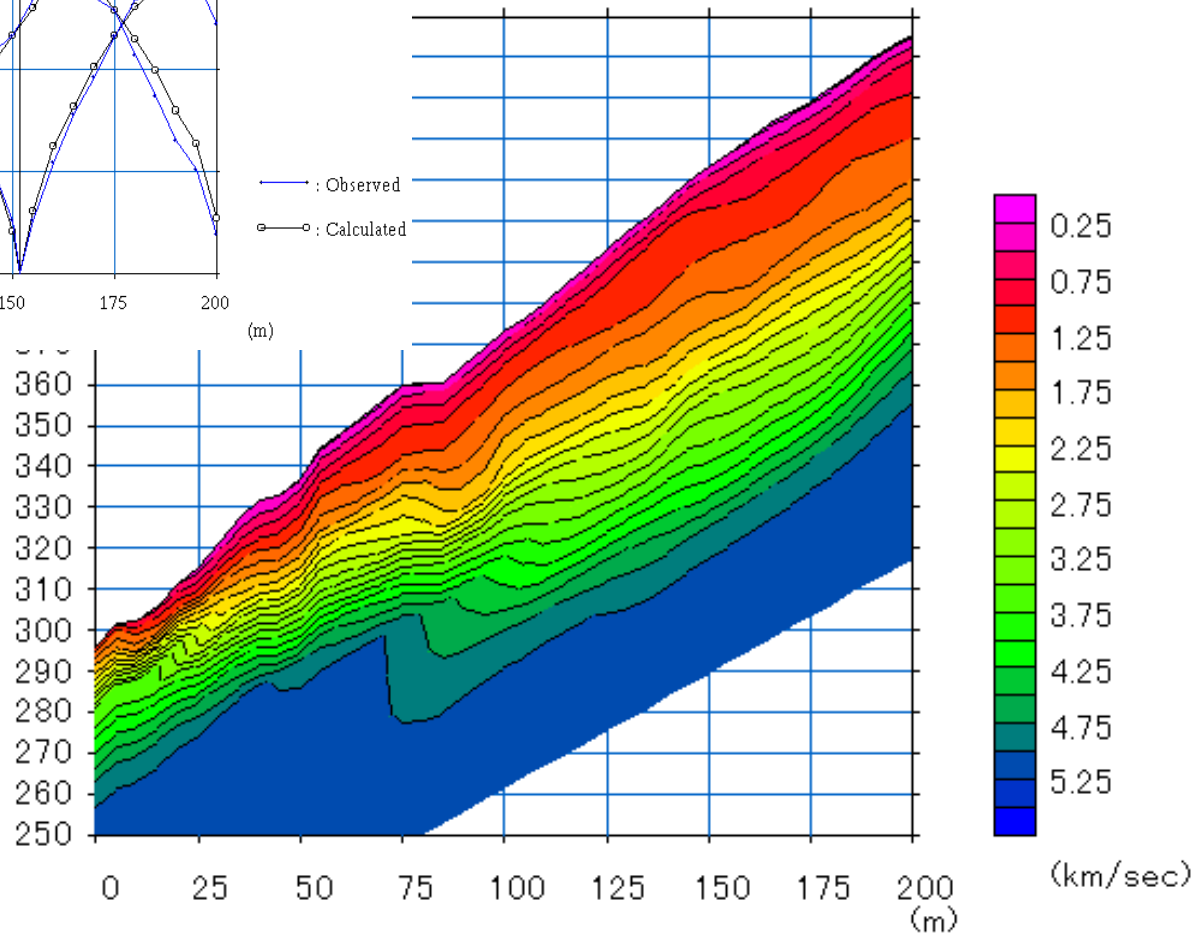
Survey location



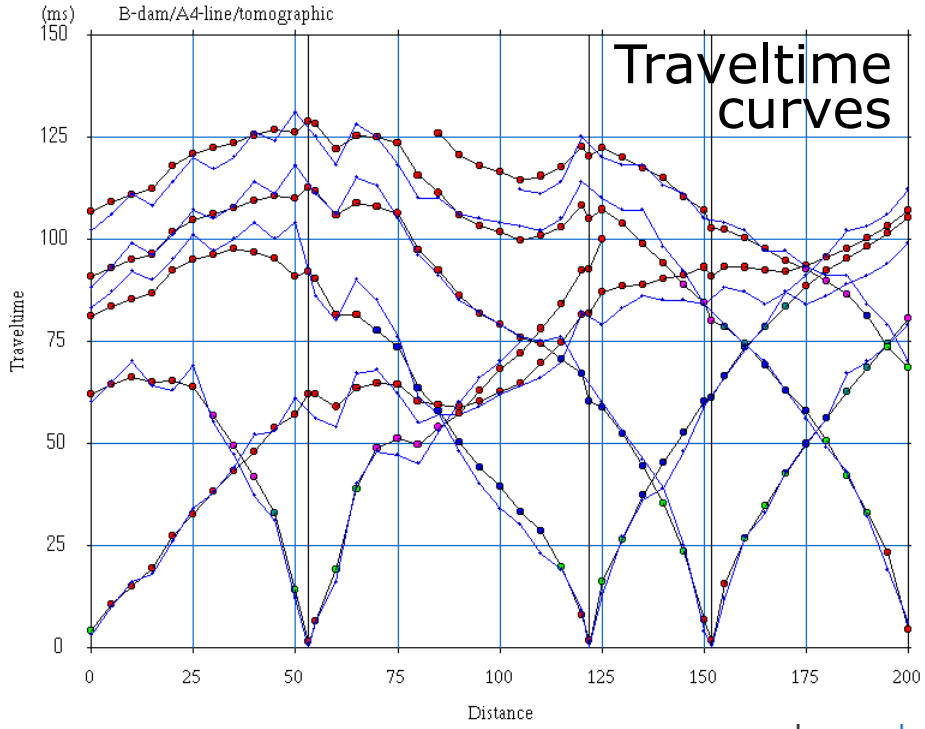
B-dam / A3-line
(outside rock-mass sliding area)



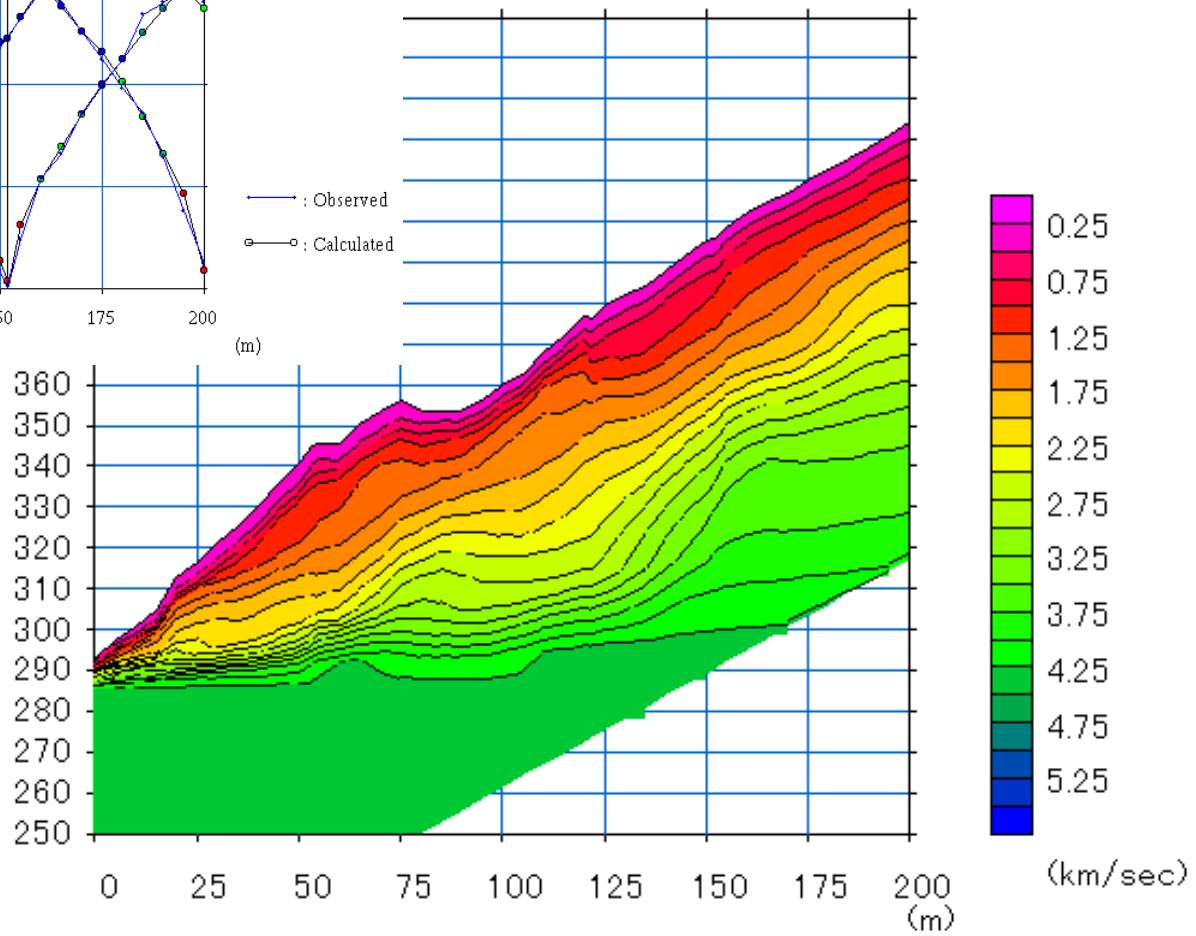
reanalyzed velocity structure



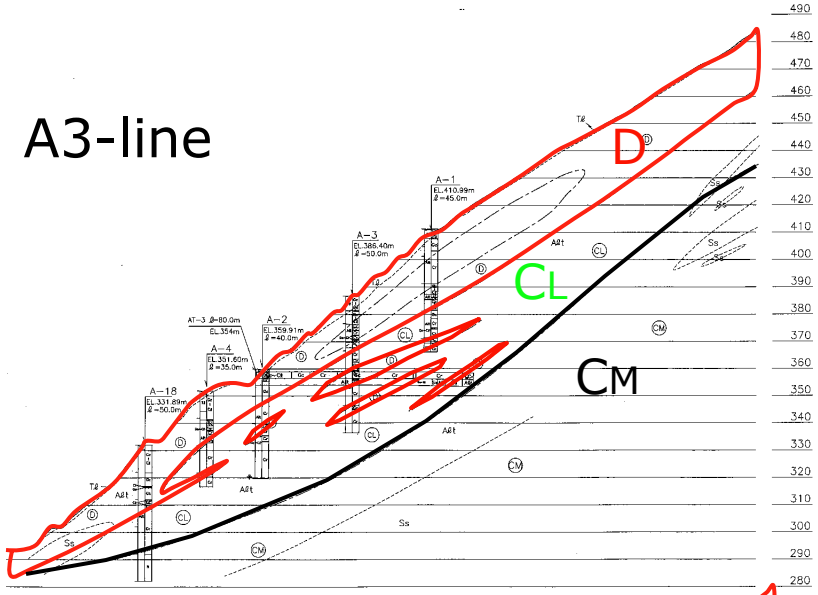
B-dam / A4-line
(on the rock-mass sliding area)



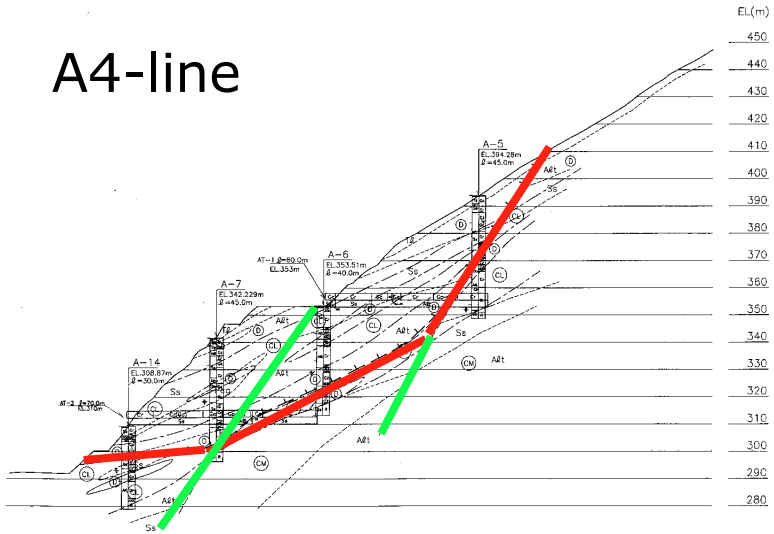
reanalyzed velocity structure

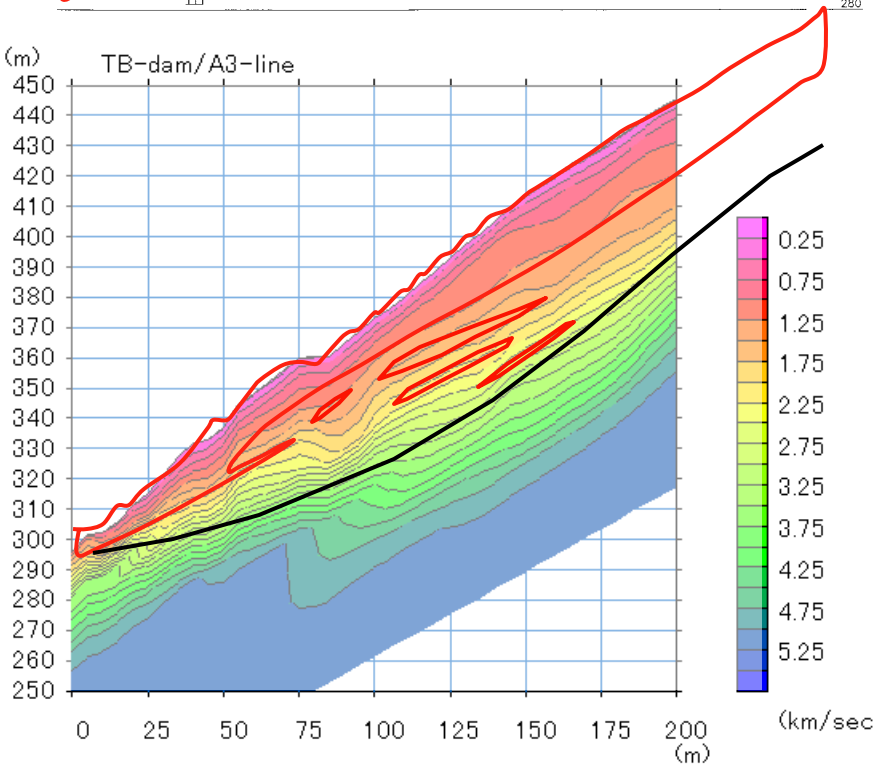


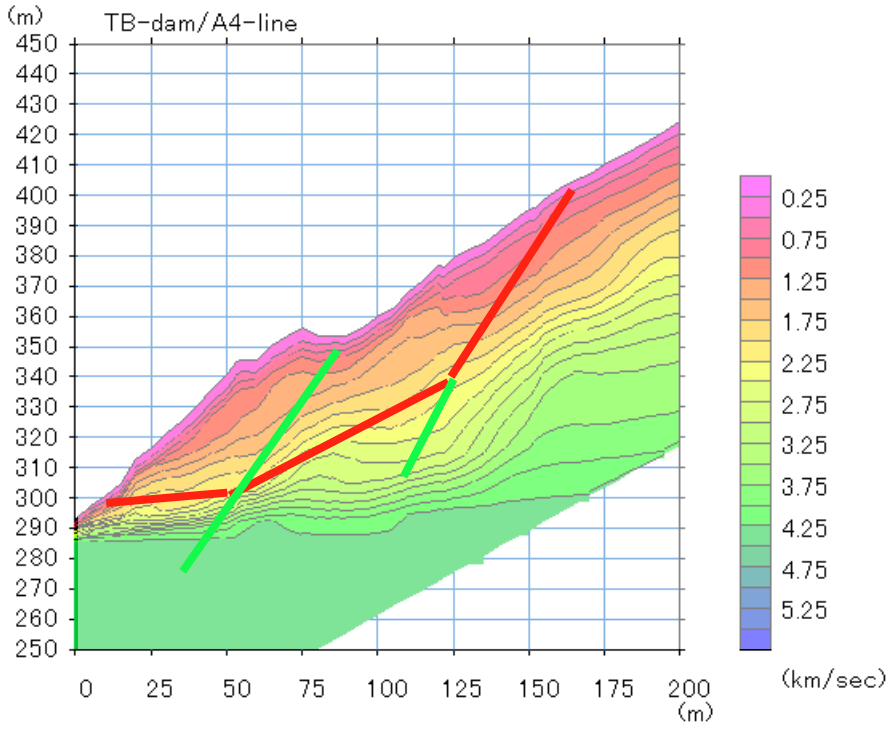
A3-line



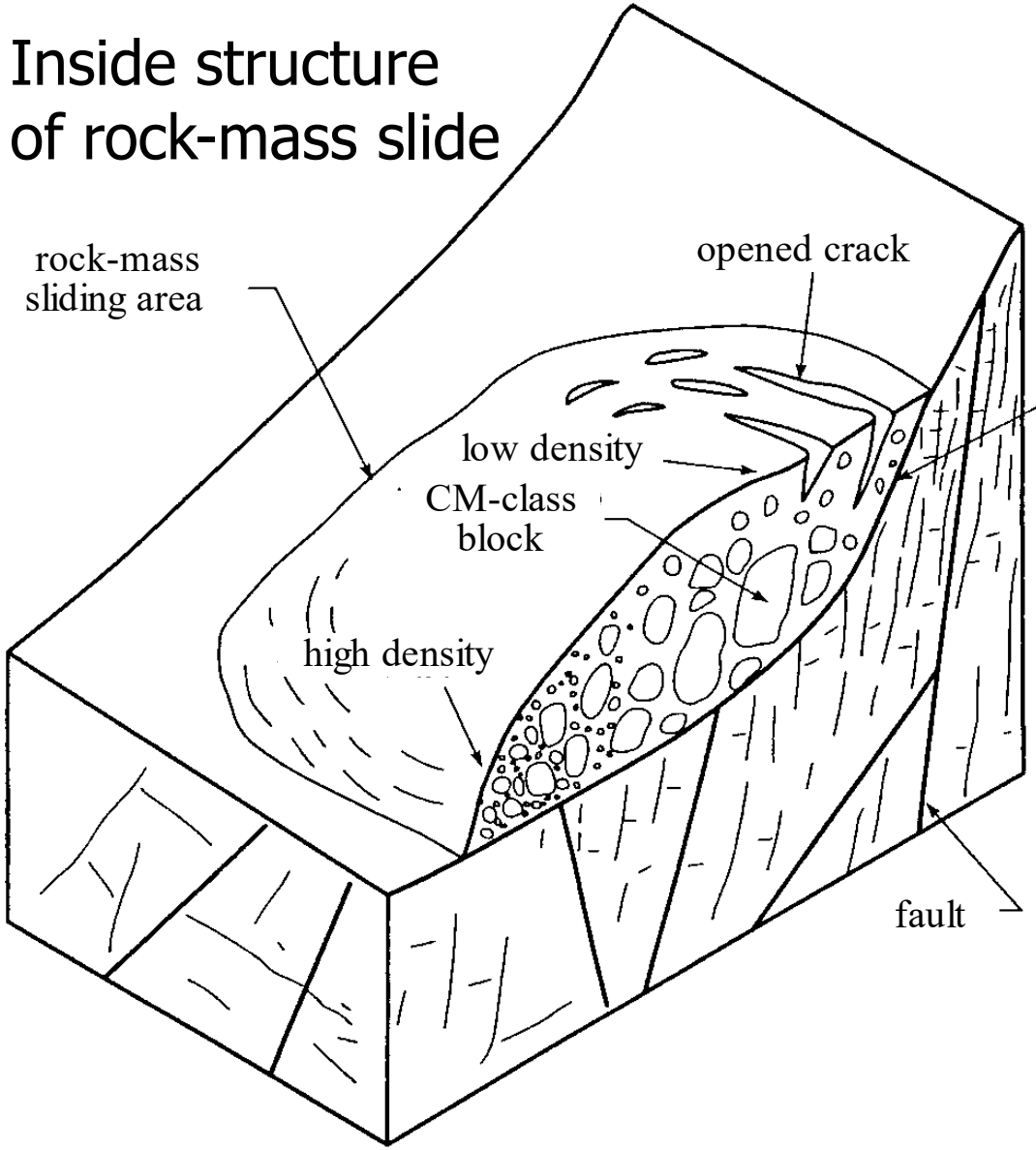
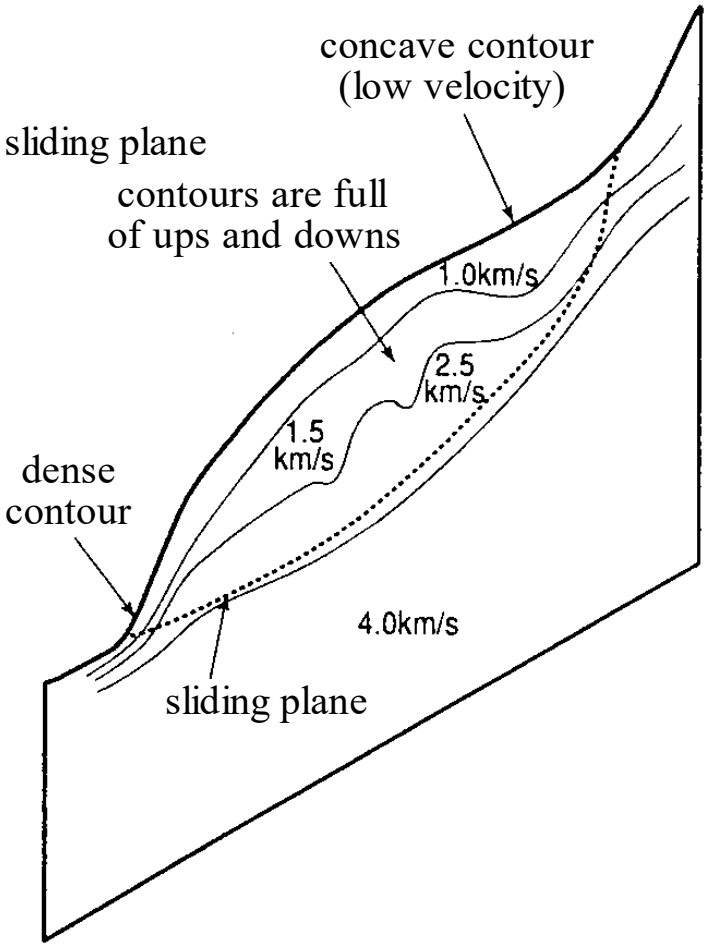
A4-line







Inside structure of rock-mass slide



Velocity distribution from refraction tomography